Section 7.3: Trigonometric Substitution

Comparison of two integrals.

$$\int x\sqrt{1-x^2}\ dx = \int \frac{-1}{2}u^{1/2}du = \frac{-1}{2} * \frac{2}{3}u^{3/2} + C = \frac{-1}{3}(1-x^2)^{3/2} + C$$

$u = 1 - x^2$ $\qquad\qquad$ $\frac{-1}{2}du = x\ dx$

$$\int \sqrt{1-x^2}\ dx$$

Examine: $\sqrt{1-x^2}$

Some useful integrals.

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\ dx = \ln|\csc x - \cot x| + C$$

$$\int \sec^3 x\ dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \csc^3 x\ dx = \frac{-1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$$

Expression	Substitution	Identity
$\sqrt{a^2-x^2}$	$x = a\sin\theta,\ \ \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$	$1-\sin^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta,\ \ \frac{-\pi}{2} < \theta < \frac{\pi}{2}$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta,\ \ 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Compute these integrals

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

1. $\int \sqrt{16 - x^2}\, dx$

$$\cos^2\theta = 1 - \sin^2\theta$$

2. $\displaystyle\int \frac{1}{(x^2-9)^{3/2}}\,dx$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

3. $\displaystyle\int \frac{1}{x^2\sqrt{16-9x^2}}\,dx$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

4. $\displaystyle\int \frac{1}{x^2 + A^2}\ dx$

Review of completing the square:

$x^2 + 8x =$

$4x^2 + 24x + 11 =$

Compute

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

5. $\displaystyle\int \frac{x}{\sqrt{4x - x^2}}\,dx$