Section 7.4: Integration of Rational Functions by Partial Fractions

$$\frac{3}{x+2}+\frac{4}{x+5}=\frac{3(x+5)+4(x+2)}{(x+2)(x+5)}=\frac{7x+23}{x^2+7x+10}$$

$$\int \frac{7x+23}{x^2+7x+10}\,dx=\int \frac{3}{x+2}+\frac{4}{x+5}\,dx=$$

A rational function is a function of the form $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are polynomials. The degree of a polynomial is the highest power of the variable.

NOTE: To integrate a rational function, $\frac{P(x)}{Q(x)}$, with the partial fraction method, you MUST HAVE the degree $P(x) <$ degree $Q(x)$. If this is not the case then use long division(or some other method) to find $J(x)$ and $K(x)$ so that

$$\frac{P(x)}{Q(x)}=J(x)+\frac{K(x)}{Q(x)}$$

Method of Integration by Partial Fractions:
0) Do long division(or other algebra manipulation) if degree $P(x) \geq$ degree $Q(x)$.
1) Factor the denominator completely
2) Decompose the fraction
3) Solve for the constants in the decomposition
4) Integrate the new fractions

Example: Compute these Integrals.

A) $\displaystyle\int \frac{x^3+2x^2-5}{x+1}\,dx$

B) $\displaystyle\int \frac{x}{x+5}\ dx$

C) $\displaystyle\int \frac{x^3+3x-5}{x^2+1}\ dx$

Example: Write out the partial fraction decomposition. Do not determine the numerical values of the coefficients.

A) $\dfrac{-3x+20}{x^3+3x^2-10x}$

B) $\dfrac{x-3}{x(x+1)^3(x^2+5)}$

C) $\dfrac{x^2+2}{x^3(x^2-9)(x^2+16)^2}$

Example: Compute these integrals.

A) $\displaystyle\int \frac{-3x+20}{x^3+3x^2-10x}\,dx$

A) Method 2: $\displaystyle\int \frac{-3x+20}{x^3+3x^2-10x}\,dx$

B) $\displaystyle\int \frac{x+2}{x^3+2x}\,dx$

C) $\displaystyle\int \frac{15x+5}{(x+2)^2(x^2+1)}\,dx$