Section 11.4: The Comparison Tests

Note: In this section all series have positive terms.

The Comparison Test (Strict Comparison): Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(a) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

(b) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Example: Do these series converge or diverge?

A) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{5n^3 + n^2 + 1}$

B) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{2n+1}}{7^n + 5}$

C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n - 2}$

Limit Comparison Test(LCT): Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = L \geq 0$

If $L > 0$ then both series converge or both series diverge.

If $L = 0$ and $\sum b_n$ converge, then $\sum a_n$ converge.

If $L = \infty$ and $\sum b_n$ diverge, then $\sum a_n$ diverge.

(Note: This test is slightly different that the test given in the book.)

Example: Do these series converge or diverge?

A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n - 2}$

B) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{\sqrt{n^2+2n}-7}$

C) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

D) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+5n}{2^n(n^2+1)}$

E) $\displaystyle\sum_{n=2}^{\infty} \frac{5+cos(n)}{\sqrt{n-1}}$