Section 11.5: Alternating Series

An **alternating series** is a series whose terms are alternately positive and negative. The general term, a_n, is of the form $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$ or $a_n = (-1)^{n-1} b_n$, where b_n is a positive number.

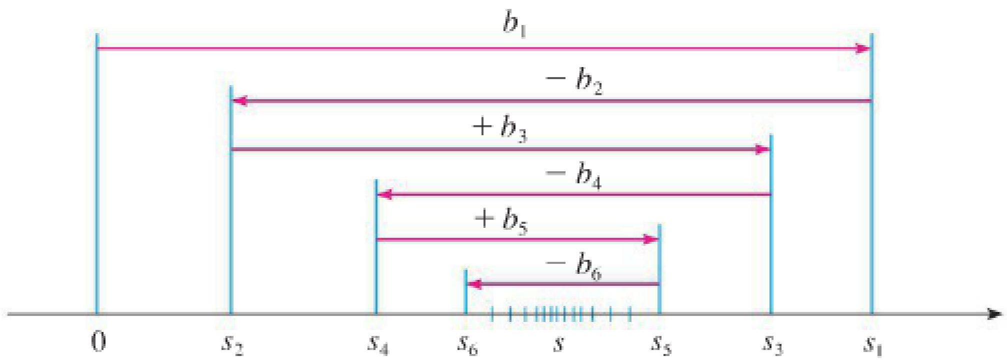
The Alternating Series Test (AST): If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{(n-1)} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \ldots.$$

with $b_n > 0$ satisfies:

(1) $b_{n+1} \le b_n$ for all n and (2) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.



Note: The Alternating Series Test does not tell us if a series will diverge.

Example: Does this series converge or diverge? $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alternating Series Estimation Theorem: If $s = \displaystyle\sum_{n=1}^{\infty} (-1)^{(n-1)} b_n$ is the sum of an alternating series that satisfies:

(a) $0 < b_{n+1} \le b_n$ and (b) $\lim_{n\to\infty} b_n = 0$

then $|R_n| = |s - s_n| \le b_{n+1}$

Example: Find a bound on R_4 for the series: $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n}$

Example: Do these series converge or diverge?

A) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n^2}\right)$

B) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{n^2}$

C) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{4^n}$

D) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2+1}$

Example: Determine if the series converges or diverges. If it converges find a bound for the error of s_6, i.e. R_6

$$\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$$

Example: What is the smallest number of terms we must use to approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ so that the error is less than $\dfrac{1}{120}$.

Example: What is the minimum number of terms needed so that the sum of this series is correct to 3 decimal places? i.e. error < 0.0005

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)!}$$

Example: Given that the alternating series $\sum_{n=1}^{\infty} (-1)^{(n-1)} b_n$ converges. Is the sum of the first 47 terms, s_{47}, an overestimate or an underestimate for the total sum?