Exam 2 Information

You are encouraged to check this document to make sure that I did not accidentally have typos in any of the formulas.

Chapter 3

- Annuity immediate

  payment made at end of period

  n = number of payments

  i = rate per period

  PV $= a_{\overline{n}|} = \dfrac{1-v^n}{i}$

  FV $s_{\overline{n}|} = \dfrac{(1+i)^n - 1}{i}$

  $a_{\overline{n}|} = s_{\overline{n}|} v^n$

  $s_{\overline{n}|} = a_{\overline{n}|}(1+i)^n$

- Annuity due

  payment made at beging of period

  n = number of payments

  i = rate per period

  PV $= \ddot{a}_{\overline{n}|} = \dfrac{1-v^n}{d}$

  FV $= \ddot{s}_{\overline{n}|} = \dfrac{(1+i)^n - 1}{d}$

  $\ddot{a}_{\overline{n}|} = \ddot{s}_{\overline{n}|} v^n$

  $\ddot{s}_{\overline{n}|} = \ddot{a}_{\overline{n}|}(1+i)^n$

- perpetuity

  $a_{\overline{\infty}|} = \dfrac{1}{i}$

  $\ddot{a}_{\overline{\infty}|} = \dfrac{1}{d}$

- relationships

  $\ddot{a}_{\overline{n}|} = (1+i) a_{\overline{n}|} = 1 + a_{\overline{n-1}|}$

  $\ddot{s}_{\overline{n}|} = (1+i) s_{\overline{n}|} = s_{\overline{n+1}|} - 1$

- Annuity value one any date
- deferred annuity
- unknown time
  - ballon payment
  - drop paymnet
- annuity with unknown rate of interest
- Annuit with varying interest
  - portfolio method
  - yield curve method

Chapter 4

- Annuities with differing payment and interest conversion periods.

  Method 1: convert interest conversion period to coincide to the payment period. Solve using chapter 3 methods.

  Method 2: Algebraic analysis.

- **Annuity-immediate payable <u>less</u> frequently than interest conversions.**
  A payment period had more than one interest conversion periods.

  i is the rate per interest conversion period

  k is the number of interest conversions periods in one payment period

  n is the total number of interest conversion periods

  pays 1 at the end of each k interest periods

  $PV = \dfrac{a_{\overline{n}|i}}{s_{\overline{k}|i}}$

  $FV = PV(1+i)^n = \dfrac{s_{\overline{n}|i}}{s_{\overline{k}|i}}$

  present value of perpetuity $= \dfrac{1}{i s_{\overline{k}|i}}$

- **Annuity-due payable <u>less</u> frequently than interest conversions.**
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  i is the rate per interest conversion period

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  n is the total number of interest conversion periods

  pays 1 at the beginning of each k interest periods

  $PV = \dfrac{a_{\overline{n}|i}}{a_{\overline{k}|i}} = \dfrac{\ddot{a}_{\overline{n}|i}}{\ddot{a}_{\overline{k}|i}}$

  $FV = PV(1+i)^n = \dfrac{s_{\overline{n}|i}}{a_{\overline{k}|i}} = \dfrac{\ddot{s}_{\overline{n}|i}}{\ddot{a}_{\overline{k}|i}}$

  present value of perpetuity $= \dfrac{1}{d \ddot{a}_{\overline{k}|i}} = \dfrac{1}{i a_{\overline{k}|i}}$

  alternate formula for present value:
  $\dfrac{a_{\overline{n}|i}}{s_{\overline{k}|i}}(1+i)^k = \dfrac{a_{\overline{n}|i}}{s_{\overline{k}|i} v^k} = \dfrac{a_{\overline{n}|i}}{a_{\overline{k}|i}}$

  alternate formula for future value:
  $\dfrac{s_{\overline{n}|i}}{s_{\overline{k}|i}}(1+i)^k = \dfrac{s_{\overline{n}|i}}{s_{\overline{k}|i} v^k} = \dfrac{s_{\overline{n}|i}}{a_{\overline{k}|i}}$

- **Annuity-immediate payable <u>more</u> frequently than interest conversions.**
Interest is done annually(annual effective) and have multiple payment peroids in a year.

m is the number of payments in one interest conversion period.

n is the total number of interest conversion periods

i is the rate per interest conversion period(assumed annually)

$i^{(m)} = m\left[(1+i)^{1/m} - 1\right]$

total amount paid is 1 in the interest conversion period where each payment period is paid 1/m.

$PV = a^{(m)}_{\overline{n}|i} = \dfrac{1-v^n}{i^{(m)}}$

$FV = s^{(m)}_{\overline{n}|i} = a^{(m)}_{\overline{n}|i}(1+i)^n = \dfrac{(1+i)^n - 1}{i^{(m)}}$

present value of perpetuity $= a^{(m)}_{\overline{\infty}|i} = \dfrac{1}{i^{(m)}}$

- **Annuity-due payable <u>more</u> frequently than interest conversions.**
Interest is done annually(annual effective) and have multiple payment peroids in a year.

m is the number of payments in one interest conversion period.

n is the total number of interest conversion periods

i is the rate per interest conversion period (asumed anually)

$d^{(m)} = m\left[1 - (1+i)^{-1/m}\right]$

total amount paid is 1 in the interest conversion period where each payment period is paid 1/m.

$PV = \ddot{a}^{(m)}_{\overline{n}|i} = \dfrac{1-v^n}{d^{(m)}}$

$FV = \ddot{s}^{(m)}_{\overline{n}|i} = \ddot{a}^{(m)}_{\overline{n}|i}(1+i)^n = \dfrac{(1+i)^n - 1}{d^{(m)}}$

present value of perpetuity $= \ddot{a}^{(m)}_{\overline{\infty}|i} = \dfrac{1}{d^{(m)}}$

- **Continuous Annuity**

n is the number of interest conversion periods

total amount paid during each interest conversion period = 1

$$\bar{a}_{\overline{n}|} = \int_0^n v^t dt = \frac{v^n - 1}{\ln(v)} = \frac{1-v^n}{\ln(1+i)} = \frac{1-v^n}{\delta} = \frac{1-e^{-\delta n}}{\delta}$$

$$\bar{s}_{\overline{n}|} = \int_0^n (1+i)^t dt = \frac{(1+i)^n - 1}{\ln(1+i)} = \frac{(1+i)^n - 1}{\delta} = \frac{e^{\delta n} - 1}{\delta}$$

present value of perpetuity $= \lim_{n\to\infty} \bar{a}_{\overline{n}|}$

- **Annuity-immediate with payments varying in arithmetic progression**

n is the number of periods

i is the rate per interest conversion period

first payment P and increase by Q.

payment periods coincide with interest conversion periods.

$\text{PV} = Pa_{\overline{n}|} + Q\dfrac{a_{\overline{n}|} - nv^n}{i}$

$\text{FV} = PV(1+i)^n = Ps_{\overline{n}|} + Q\dfrac{s_{\overline{n}|} - n}{i}$

Perpetuity: $P > 0$ and $Q > 0$

PV$= \dfrac{P}{i} + \dfrac{Q}{i^2}$

<u>Increasing annuity</u>: P=Q = 1

$$(Ia)_{\overline{n}|} = \frac{a_{\overline{n}|} * (1+i) - nv^n}{i} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$(Is)_{\overline{n}|} = \frac{s_{\overline{n}|} * (1+i) - n}{i} = \frac{\ddot{s}_{\overline{n}|} - n}{i}$$

<u>decreasing annuity</u>: $P = n$ and $Q = -1$

$$(Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

$$(Ds)_{\overline{n}|} = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$$

- **Annuity-due with payments varying in arithmetic progression**

  n is the number of periods

  i is the rate per interest conversion period

  first payment P and increase by Q.

  payment periods coincide with interest conversion periods.

  $$\text{PV} = P\ddot{a}_{\overline{n}|} + Q\frac{a_{\overline{n}|} - nv^n}{d}$$

  $$\text{FV} = PV(1+i)^n = P\ddot{s}_{\overline{n}|} + Q\frac{s_{\overline{n}|} - n}{d}$$

  Perpetuity: $P > 0$ and $Q > 0$

  $$\text{PV}= \frac{P}{d} + \frac{Q}{id}$$

  Increasing annuity: P=Q = 1

  $$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

  $$(I\ddot{s})_{\overline{n}|} = (1+i)^n(I\ddot{a})_{\overline{n}|} = \frac{\ddot{s}_{\overline{n}|} - n}{d}$$

  decreasing annuity: $P = n$ and $Q = -1$

  $$(D\ddot{a})_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{d}$$

  $$(D\ddot{s})_{\overline{n}|} = (1+i)^n(D\ddot{a})_{\overline{n}|} = \frac{n(1+i)^n - s_{\overline{n}|}}{d}$$

- **Annuity-immediate with payments varying in geometric progression**

  n is the number of periods

  i is the rate per interest per interest conversion period

  first payment is 1 and successive payments increase/decrease by k%. growth ratio of $(1+k)$.

  Method 1: working the problem as regular

  $$PV = \frac{1 - \left(\frac{1+k}{1+i}\right)^n}{(1+i)-(1+k)} = \frac{1 - \left(\frac{1+k}{1+i}\right)^n}{i-k}$$

  $$FV = PV(1+i)^n = \frac{(1+i)^n - (1+k)^n}{i-k}$$

  Perpetuity: $k < i$

  $$PV = \frac{1}{i-k}$$

  Method 2: working the problem with an adjusted rate per period.

  adjusted rate: $1 + j = \dfrac{1+i}{1+k}$

  $$PV = \frac{1}{1+k}\ a_{\overline{n}|j}$$

  $$FV = PV(1+i)^n = \frac{(1+i)^n - (1+k)^n}{i-k}$$

  Perpetuity: $k < i$

  $$PV = \frac{1}{1+k} * \frac{1}{j}$$

- **Annuity-due with payments varying in geometric progression**

  n is the number of periods

  i is the rate per interest per interest conversion period

  first payment is 1 and successive payments increase/decrease by k%. growth ratio of $(1+k)$.

  $PV = (1+i)*$ PV of annuity-immediate

  With adjusted rate per period

  $$PV = \frac{1}{1+k}\ a_{\overline{n}|j}(1+i) = \frac{1+i}{1+k}\ a_{\overline{n}|j}$$

  $$= (1+j)\ a_{\overline{n}|j} = \ddot{a}_{\overline{n}|j}$$

Any additional topic/information covered in these chapters.