Chapter 3: Basic Annuities

Section 3.1: Introduction

An **annuity** may be defined as a series of payments made at equal intervals of time.

An **annuity-certain** is an annuity such that payments are certain to be made for a fixed period of time (the **term** of the annuity).

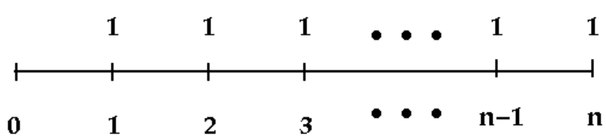
A **contingent annuity** is an annuity under which the payments are not certain. i.e. payments from a pension plan for the life of a retiree.

The interval between annuity payments is called the **payment period**. This chapter considers annuities where the payment period and the interest conversion period are equal and coincide.

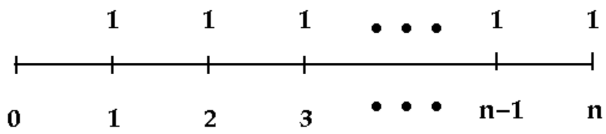
Section 3.2: Annuity-Immediate

An annuity under which payments are made at the end of each payment period for n periods, where n is a positive integer, is called an **annuity-immediate** or an **ordinary annuity** or just an **annuity**.

Consider the annuity where payments of \$1 are made at the end of the period for n periods.



- The present value (PV) of the annuity is denoted by $a_{\overline{n}|}$ or $a_{\overline{n}|i}$.



- The accumulated value (FV) of the annuity is denoted by $s_{\overline{n}|}$ or $s_{\overline{n}|i}$.

Relationship between $s_{\overline{n}|}$ and $a_{\overline{n}|}$

Geometric Progression/Geometric Series

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k$$

Example: David will receive payments of \$50 at the end of each month for the next 8 years. Assume $i^{(12)} = 9\%$

(a) Find the present value of this annuity.

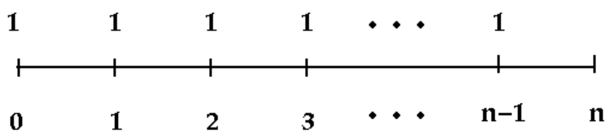
(b) Find the accumulated value of this annuity.

Example: How much should be deposited at the end of each quarter so that at the end of 15 years the account balance is \$75,000? Assume an annual effective rate of interest of 6.14%.

Example: Bob invests a \$15,000 gift at nominal rate of 6% compounded quarterly. How much can be withdrawn at the end of every quarter to use up the fund exactly at the end of 6 years of college?

Section 3.3: Annuity-Due

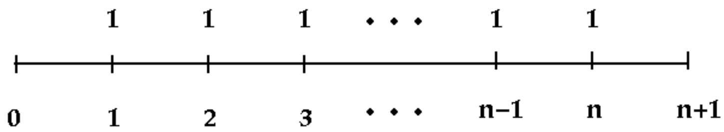
An **annuity-due** is an annuity for which payments are made at the beginning of the period.



- The present value (PV) of the annuity-due is denoted by $\ddot{a}_{\overline{n}|}$ or $\ddot{a}_{\overline{n}|i}$.

- The accumulated value (FV) of the annuity-due is denoted by $\ddot{s}_{\overline{n}|}$ or $\ddot{s}_{\overline{n}|i}$.

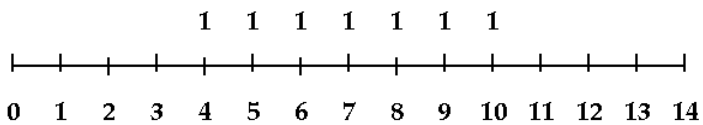
Relationship between $\ddot{s}_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}$, $s_{\overline{n}|}$, and $a_{\overline{n}|}$



Example: Sam wishes to accumulate \$30,000 in an account in 7 years. He will make deposits semiannually with the first deposit at time 0 and the last deposit at time 6.5. How large should the deposit be if the account earns a nominal rate of 8% compounded semianuualy.

Section 3.4: Annuity Values on any Date

Example: Suppose 7 payments of 1 are made at the end of the 4th through 10th periods, inclusive.



Find the value of the annuity

(a) at the end of the 1st period.

Note: This in an example of a deferred annuity, since payments only commence after a deferred period.
Notation: ${}_{m|}a_{\overline{n}|}$ is n payments deffered after m periods.

(b) at the end of the 14th period.

(c) At the end of the 7th period.