## Chapter 5: Amortization schedules ad Sinking Funds

### Section 5.4: Sinking Funds

The borrower repays the lender by means of a single lump-sum payment at the end of the term of the loan. It is assumed that the borrower makes periodic payments into a fund, called a "sinking fund," which will accumulate to the amount of the loan to be repaid at the end of the term of the loan. Sometimes the fund is created so that it will be sufficient to repay the loan.

With a sinking fund, interest payments are *usually* paid to the lender in a periodic manner on the entire amount of the loan These interest payments are sometimes called the **service on the loan**.

If the rate of interest paid on the loan equals the rate of interest earned on the sinking fund and the sinking fund is designed to exactly pay off the loan (at the end of the term), then the sinking fund method is equivalent to the amortization method.

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Example A loan of $5000 is being repaid in 4 years via the sinking fund method. Write a sinking fund schedule for the repayment of this loan assuming annual payments into the sinking fund. Assume the interest charged on the loan and the interest the sinking fund earns is an annual rate of 6.09%.

| Period | Interest Paid | Sinking Fund Deposit | Interest Earned on Sinking Fund | Amount in Sinking Fund | Net amount of the Loan |
|--------|---------------|----------------------|--------------------------------|------------------------|------------------------|
| 0 | — | — | — | 0 | 5000 |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

**Because the interest rate were equal**

1) The total amount paid in the sinking fund method equals the total amount paid in amortization method.

2) Net interest paid in the sinking fund method equals the amount of interest paid in the amortization method.

3) The annual amount added to the sinking fund (deposit and interest) equals the principal repaid in each period of the amortization method.

4) Net amount of the loan in sinking fund method equals the outstanding balance in the amortization method.

Amortization Information

| Period | Payment | Interest Paid | Principal Repaid | Outstanding Balance |
|--------|---------|---------------|------------------|---------------------|
| 0 | — | — | — | 5000 |
| 1 | 1445.93 | 304.50 | 1141.43 | 3858.57 |
| 2 | 1445.93 | 234.99 | 1210.94 | 2647.63 |
| 3 | 1445.93 | 161.24 | 1284.69 | 1362.94 |
| 4 | 1445.93 | 83.00 | 1362.93 | 0.01 |
| Totals | 5783.72 | 783.73 | 4,999.99 | |

Sinking fund Information

| Period | Interest Paid | Sinking Fund Deposit | Interest Earned | Amount in Fund | Net amount of the Loan |
|--------|---------------|----------------------|-----------------|----------------|------------------------|
| 0 | — | — | — | 0 | 5000 |
| 1 | 304.50 | 1141.43 | 0 | 1141.43 | 3858.57 |
| 2 | 304.50 | 1141.43 | 69.51 | 2352.37 | 2647.63 |
| 3 | 304.50 | 1141.43 | 143.26 | 3637.06 | 1362.94 |
| 4 | 304.50 | 1141.43 | 221.50 | 4999.99 | .01 |
| Total | 1218.00 | 4565.72 | 434.27 | | |

## Sinking Fund Method with Different Interest Rates

Suppose a loan of $5000 is to be repaid in 4 periods with the sinking fund method.

$i = 10\%$ rate of interest paid on loan
$j = 6.09\%$ rate of interest earned on sinking fund.

| Period | Interest Paid | Sinking Fund Deposit | Interest Earned on Sinking Fund | Amount in Sinking Fund | Net amount of the Loan |
|--------|---------------|----------------------|---------------------------------|------------------------|------------------------|
| 0      | —             | —                    | —                               | —                      | 5000                   |
| 1      |               | 1141.43              | 0                               | 1141.43                | 3858.57                |
| 2      |               | 1141.43              | 69.51                           | 2352.37                | 2647.63                |
| 3      |               | 1141.43              | 143.26                          | 3637.06                | 1362.94                |
| 4      |               | 1141.43              | 221.50                          | 4999.99                | .01                    |

If you wanted an amortization method that was equivalent, what interest rate would you need?

Example: A payment of $36,000 is made at the end of each year for 31 years to repay a loan of $400,000. If the borrower replaces the capital by means of a sinking fund earning 3% effective, find the effective rate paid to the lender on the loan.

Example: Bob borrows $5000 for 10 years at 10% convertible quarterly. Bob does not pay interest currently and will pay all accrued interest at the end of 10 years together with the principal. Find the annual sinking fund deposit necessary to liquidate the loan at the end of 10 years if the sinking fund earns 7% convertible semiannually.

Example: A loan of $20,000 is to amortized with monthly payments, at the end of each month, of $400 until the loan is paid off. The final payment will be some amount less than the monthly payment. $i^{(12)} = 6\%$

A) The amount of interest paid in the 4th year.

B) The amount of principal paid in the 4th year.