Section 2.3: Additional Problems Solutions

1. $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1} = \lim_{x\to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \dfrac{x^2+x+1}{x+1} = \dfrac{3}{2}$

2. $\lim_{x\to -2} \dfrac{3x^2+2x-8}{5x^2+17x+14} = \lim_{x\to -2} \dfrac{(x+2)(3x-4)}{(x+2)(5x+7)} = \lim_{x\to -2} \dfrac{3x-4}{5x+7} = \dfrac{-10}{-3} = \dfrac{10}{3}$

3. $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{7}{4x^2+7x}\right) = \lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{7}{x(4x+7)}\right) = \lim_{x\to 0} \left(\dfrac{4x+7}{x(4x+7)} - \dfrac{7}{x(4x+7)}\right) =$

   $\lim_{x\to 0} \dfrac{4x}{x(4x+7)} = \lim_{x\to 0} \dfrac{4}{4x+7} = \dfrac{4}{7}$

4. If we want $\lim_{x\to 2} f(x) = 10$, this means that we must have $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 10$

   So now check the one-sided limits.

   <u>limit from the left</u>

   $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 3x+4 = 10$

   <u>limit from the right</u>

   $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 4x^2+A = 16+A$

   Notice the left handed limit is 10 and it does not involve the variable A. To get the right handed limit to be 10 we need

   $10 = 16 + A$ or $A = -6$.

5. $\lim_{x\to -5} \dfrac{3x^2+17x+10}{2x^3+9x^2-5x} = \lim_{x\to -5} \dfrac{(3x+2)(x+5)}{x(2x-1)(x+5)} = \lim_{x\to -5} \dfrac{3x+2}{x(2x-1)} = \dfrac{-13}{55}$

6. $\lim_{x\to -4} \dfrac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to -4} \dfrac{\sqrt{x^2+9}-5}{x+4} * \dfrac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} = \lim_{x\to -4} \dfrac{(x^2+9)-5^2}{(x+4)(\sqrt{x^2+9}+5)} =$

   $\lim_{x\to -4} \dfrac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x\to -4} \dfrac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x\to -4} \dfrac{(x+4)(x-4)}{(x+4)(\sqrt{x^2+9}+5)} =$

   $\lim_{x\to -4} \dfrac{x-4}{\sqrt{x^2+9}+5} = \dfrac{-8}{10}$