Section 3.2: Additional Problems Solutions

1. Find the equation of the tangent line at $x = 2$ for $f(x) = \dfrac{x}{x-1}$

   The point that we want the tangent line at is $(2, f(2))$ or $(2, 2)$.

   Now find the slope of the tangent line.

   $f'(x) = \frac{(x-1)*1-x*(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

   $m_{tan} = f'(2) = \frac{-1}{(2-1)^2} = -1$

   Tangent line: $y - 2 = -1(x - 2)$

   The normal line and the tangent line go through the same point. The only thing different is the slope fo the normal line is perpendicular to the slope fo the tangent line.

   $m_{normal} = \dfrac{-1}{m_{tan}} = \dfrac{-1}{-1} = 1$

   normal line: $y - 2 = 1(x - 2)$

2. Find the value(s) of x where the tangent line to $f(x) = \dfrac{x}{x-1}$ will go through the point $(6, -2)$. Show the work that verifies your answers.
   Notice that the point $(6, -2)$ is not on the graph of the function.

   Let the $x = A$ the value of x where the tangent line at that point will go through $(6, -2)$.

   we will need $f(A)$ and $f'(A)$.

   $f'(x) = \frac{(x-1)*1-x*(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

   $f'(A) = \dfrac{-1}{(A-1)^2}$ and $f(A) = \dfrac{A}{A-1}$

   Now $y - f(A) = f'(A)(x - A)$ is the equation of the tangent line.

   $y - \dfrac{A}{A-1} = \dfrac{-1}{(A-1)^2}(x - A)$

   now plug in the point for the x and y.

   $-2 - \dfrac{A}{A-1} = \dfrac{-1}{(A-1)^2}(6 - A)$

   now multiply both sides of the equation by $(A-1)^2$ to siplify the equation.

$-2(A-1)^2 - A(A-1) = -(6-A)$

$-2(A^2-2A+1) - A^2 + A = -6 + A$

$-2A^2 + 4A - 2 - A^2 + A = -6 + A$

$-3A^2 + 4A + 4 = 0$ or $3A^2 - 4A - 4 = 0$

$(3A+2)(A-2) = 0$

Answer: 2 and $\dfrac{-2}{3}$

3. $\dfrac{d^2y}{dx^2} = y''$ this is the second derivative.

   $y' = \dfrac{(x+7)*(2x - (x^2+5)*1}{(x+7)^2} = \dfrac{2x^2+14x-x^2-5}{(x+7)^2} = \dfrac{x^2+14x-5}{(x+7)^2}$

   since we do not know the chain rule at this time expand out the denominator.

   $y' = \dfrac{x^2+14x-5}{x^2+14x+49}$

   Answer: $y'' = \dfrac{(x^2+14x+49)*(2x+14) - (x^2+14x-5)*(2x+14)}{(x^2+14x+49)^2}$

4. $y' = (7x^6 + 12x^3)(x^8 + 7x + 1) + (x^7 + 3x^4 + 5)(8x^7 + 7)$

5. $y' = (9x^8 - 5x^{-6})(x^{-3} - 2x^{-2}) + (x^9 + x^{-5})(-3x^{-4} + 4x^{-3})$

6. $y' = (3x^2+4)e^x + (x^3 + 4x + 2)e^x$

7. $f'(x) = (3x^2 + 10x)\log_4(x) + (x^3 + 5x^2 + 1)\dfrac{1}{x\ln(4)}$

8. $y' = \dfrac{(x^5+5x^3+6)(4x^3+14x) - (x^4+7x^2+8)(5x^4+15x^2)}{(x^5+5x^3+6)^2}$

9. $y' = \dfrac{(x^4+7x^3+5)(3^x\ln(3)+7) - (3^x+7x)(4x^3+21x^2)}{(x^4+7x^3+5)^2}$

10. $f'(x) = \dfrac{(x^5+7x)[4x^3e^x + x^4e^x] - (x^4e^x)(5x^4+7)}{(x^5+7x)^2}$

11. $J'(x) = f'(x)g(x) + f(x)g'(x)$
    $J'(0) = f'(0)g(0) + f(0)g'(0)$
    $J'(0) = (-3)(3) + (1)(5)$

    Answer: $J'(0) = -4$

12. $H'(3) = -8$

13. $K'(x) = \dfrac{f(x)\left(3x^2 + \dfrac{1}{x}\right) - (x^3 + \ln(x))f'(x)}{[f(x)]^2}$

$K'(1) = \dfrac{f(1)\left(3 + \dfrac{1}{1}\right) - (1 + \ln(1))f'(1)}{[f(1)]^2}$

Answer: $K'(1) = \dfrac{2}{4} = \dfrac{1}{2}$

14. $M'(2) = \dfrac{-13e^2}{9}$

15. $y' = \dfrac{(x^2 + 12)(-1) - (-x + 2)(2x)}{(x^2 + 12)^2}$

$y' = \dfrac{x^2 - 4x - 12}{(x^2 + 12)^2}$

$0 = \dfrac{x^2 - 4x - 12}{(x^2 + 12)^2}$

Note: a fraction equal to zero means that the numerator must be zero.

$0 = x^2 - 4x - 12$

Answer: $x = -2$, $x = 6$

16. $y' = \dfrac{-2x + 6}{(x^2 - 6x + 10)^2}$

Answer: $x = 3$

17. $y' = \dfrac{-6x - 30}{(x^2 + 10x)^2}$

Answer: $x = -5$

18. $y' = \dfrac{5}{(x + 4)^2}$

$5 = \dfrac{5}{(x + 4)^2}$

Now cross multiply, i.e. multiply both sides by $(x + 4)^2$

$5(x + 4)^2 = 5$

$(x + 4)^2 = 1$

$x + 4 = \pm 1$

Answer: $x = -5$, $x = -3$

19. $x = 4$, $x = 6$