

1) Use the definition of the derivative to find  $f'(x)$  for  $f(x) = \frac{x+1}{x-1}$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{(x+h+1)(x-1)}{(x+h-1)(x-1)} - \frac{(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 - x + xh - h + x - 1 - (x^2 + xh - x + x + h - 1)}{(x+h-1)(x-1)} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + xh - h - 1 - x^2 - xh - h + 1}{(x+h-1)(x-1)} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-2h}{(x+h-1)(x-1)} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)} = \frac{-2}{(x-1)^2} = f'(x)
 \end{aligned}$$