Challenge 4.8

1] $\lim_{x\to 0} \frac{\sin(x) + e^x - 1}{x} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to 0} \frac{\cos(x) + e^x}{1} = \cos(0) + e^0 = 2$

2] $\lim_{x\to 0^+} (1+5x)^{\frac{1}{2x}} \overset{1^\infty}{=}$

let $y = (1+5x)^{\frac{1}{2x}}$

$\ln y = \frac{1}{2x} \ln(1+5x) = \frac{\ln(1+5x)}{2x}$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1+5x)}{2x} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to 0^+} \frac{\frac{5}{1+5x}}{2} = \frac{5}{2}$

$\lim_{x\to 0^+} (1+5x)^{\frac{1}{2x}} = \lim_{x\to 0} e^{\ln(1+5x)^{\frac{1}{2x}}} = \boxed{e^{5/2}}$

3] $\lim_{x\to\infty} \frac{\ln(2+e^{3x})}{\pi x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{3e^{3x}}{2+e^{3x}}}{\pi} = \lim_{x\to\infty} \frac{3e^{3x}}{2+e^{3x}} \cdot \frac{1}{\pi}$

$= \lim_{x\to\infty} \frac{3e^{3x}}{2\pi + \pi e^{3x}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{9e^{3x}}{3\pi e^{3x}} = \lim_{x\to\infty} \frac{9}{3\pi} = \boxed{\frac{3}{\pi}}$

4] $$\lim_{x\to\infty}\left(\frac{x^2}{x-1}-\frac{x^2}{x+5}\right) \overset{\infty-\infty}{=} \lim_{x\to\infty}\frac{x^2(x+5)-x^2(x-1)}{(x-1)(x+5)}$$

$$=\lim_{x\to\infty}\frac{x^3+5x^2-x^3+x^2}{x^2+4x-5} = \lim_{x\to\infty}\frac{6x^2}{x^2+4x-5} \overset{\frac{\infty}{\infty}}{=}$$

$$\lim_{x\to\infty}\frac{12x}{2x+4} \overset{L'H}{=} \lim_{x\to\infty}\frac{12}{2} = 6$$

5] $$\lim_{x\to 2^+}\frac{\ln(x-1)}{(x-2)^2} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to 2^+}\frac{\frac{1}{x-1}}{2(x-2)}$$

$$=\lim_{x\to 2^+}\frac{1}{x-1}\cdot\frac{1}{2(x-2)} = \lim_{x\to 2^+}\frac{1}{2(x-1)(x-2)} = +\infty$$