Sections 5.5: Additional Problems Solutions

Compute

1. $\int e^{kx}dx$, where k is some non-zero constant.

$u = kx$
$du = k\ dx$
$\frac{1}{k}du = dx$

$$\int e^{kx}dx = \int \frac{1}{k}e^{u}du = \frac{1}{k}e^{u} + C = \frac{1}{k}e^{kx} + C$$

Note: make your life easier by just learning this rule.

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C$$

---

2. $\int \sin(kx)dx$, where k is some non-zero constant.

$u = kx$
$du = k\ dx$
$\frac{1}{k}du = dx$

$$\int \sin(kx)dx = \int \frac{1}{k}\sin(u)du = \frac{-1}{k}\cos(u) + C = \frac{-1}{k}\cos(kx) + C$$

Note: make your life easier by just learning this rule.

$$\int \sin(kx)dx = \frac{-1}{k}\cos(kx) + C$$

---

3. $\int x^3(x^2-1)^4\ dx =$

$u = x^2 - 1$
$du = 2x\ dx$
$\frac{1}{2x}du = dx$

Notice the x^2 still needs to be replaced. So solve the initial U-sub formula for x^2 to get:

$x^2 = u + 1$

$$\int x^3(x^2-1)^4\ dx = \int x^3u^4\frac{1}{2x}du = \int \frac{1}{2}x^2u^4\ du$$
$$= \frac{1}{2}\int (u+1)u^4du = \frac{1}{2}\int u^5 + u^4\ du$$
$$= \frac{1}{2} * \left[\frac{u^6}{6} + \frac{u^5}{5}\right] + C$$

$$\int x^3(x^2-1)^4\ dx = \frac{(x^2-1)^6}{12} + \frac{(x^2-1)^5}{10} + C$$

---

4. $\int (12x^2+8)(x^3+2x)^6\ dx$

$u = x^3 + 2x$

$du = (3x^2+2)\ dx$

$$\int (12x^2+8)(x^3+2x)^6\ dx = \int 4(3x^2+2)(x^3+2x)^6\ dx$$
$$= \int 4(x^3+2x)^6(3x^2+2)\ dx = \int 4u^6du = \frac{4u^7}{7} + C$$
$$\int (12x^2+8)(x^3+2x)^6\ dx = \frac{4(x^3+2x)^7}{7} + C$$

5. $\displaystyle\int \frac{e^{2+\sqrt{x}}}{\sqrt{x}}\ dx$

$u = 2 + \sqrt{x}$

$du = \dfrac{1}{2\sqrt{x}}\ dx$

$2\sqrt{x}du = dx$

$$\int \frac{e^{2+\sqrt{x}}}{\sqrt{x}}\ dx = \int \frac{e^u}{\sqrt{x}}\ 2\sqrt{x}du = \int 2e^u\ du = 2e^u + C = 2e^{2+\sqrt{x}} + C$$

---

6. $\displaystyle\int \frac{1 - 3x^2e^{(1-x^3)}}{x + e^{(1-x^3)}}\ dx$

$u = x + e^{(1-x^3)}$

$du = (1 - 3x^2e^{1-x^3})\ dx$

$$\int \frac{1 - 3x^2e^{(1-x^3)}}{x + e^{(1-x^3)}}\ dx = \int \frac{1}{u}\ du = \ln|u| + C$$

$$\int \frac{1 - 3x^2e^{(1-x^3)}}{x + e^{(1-x^3)}}\ dx = \ln\left|x + e^{(1-x^3)}\right| + C$$

---

7. $\displaystyle\int_0^{\pi/3} \sin(\theta)\cos^2(\theta)\ d\theta =$

$$\int_0^{\pi/3} \sin(\theta)\cos^2(\theta)\ d\theta = \int_{\theta=0}^{\theta=\pi/3} -u^2\ du = \left.\frac{-u^3}{3}\right|_{\theta=0}^{\theta=\pi/3} = \left.\frac{-\cos^3\theta}{3}\right|_0^{\pi/3}$$

$u = \cos(\theta)$
$du = -\sin(theta)\ d\theta$

$$= \frac{-\cos^3(\pi/3)}{3} - \frac{-\cos^3(0)}{3} = \frac{-1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{3} = \frac{1}{3} - \frac{1}{24}$$

$$\int_0^{\pi/3} \sin(\theta)\cos^2(\theta)\ d\theta = \frac{7}{24}$$

8. If f is continuous and $\int_0^4 f(x)\ dx = 16$, find $\int_0^2 xf(x^2)\ dx$.

$u = x^2$

$du = 2x\ dx$

$\frac{1}{2}du = x\ dx$

now change the limits:
If $x = 0$ then $u = 0$.

If $x = 2$ then $u = 2^2 = 4$.

$$\int_0^2 xf(x^2)\ dx = \int_0^4 \frac{1}{2}f(u)\ du = \frac{1}{2}\int_0^4 f(u)\ du$$

Notice that $\int_0^4 f(x)\ dx = 16$, thus we know that $\int_0^4 f(u)\ du = 16$.

$$\int_0^2 xf(x^2)\ dx = \int_0^4 \frac{1}{2}f(u)\ du = \frac{1}{2}\int_0^4 f(u)\ du = \frac{1}{2} * 16 = 8$$