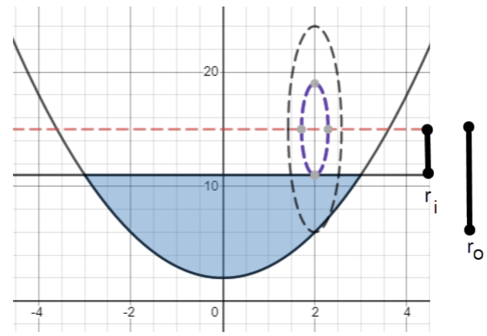
Section 6.2: Additional Problems Solutions

1. Find the volume of the solid obtained by rotating the region bounded by the given curves around $y = 15$.

   $y = x^2 + 2$
   $y = 11$

Step 1: Graph the region and find the intersection values. $x^2+2 = 11$ gives $x^2 = 9$ or $x = \pm 3$.



Notice in the picture the slice is perpendicular to the x-axis, so this will be a dx integral.

Step 2: Find the inner and outer radius.

$r_i = 15 - y = 15 - (x^2 + 2) = 13 - x^2$
$r_o = 15 - 11 = 4$

Step 3: Setup the integral and integrate.

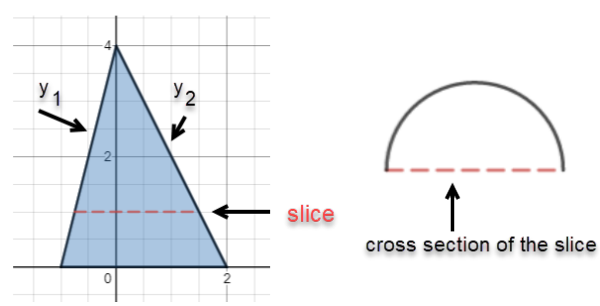
$$V = \int_{-3}^{3} \pi \left[r_o^2 - r_i^2\right]\, dx = \int_{-3}^{3} \pi \left[(13 - x^2)^2 - 4^2\right]\, dx = \int_{-3}^{3} \pi \left(169 - 26x^2 + x^4 - 16\right)\, dx$$

$$V = 2\pi \int_{0}^{3} 153 - 26x^2 + x^4 \; dx = 2\pi \left(153x - \frac{26x^3}{3} + \frac{x^5}{5}\right)\Bigg|_0^3 = 2\pi \left(152 * 3 - \frac{26 * 3^3}{3} + \frac{3^5}{5} - (0)\right)$$

$V = 547.2\pi$

2. The base of a solid is the triangle that is enclosed by the points $(0, 4)$, $(2, 0)$, and $(-1, 0)$. Cross-sections that are perpendicular to the y-axis are semicircles. Set up the integral(s) that would give the volume of the solid.

Step 1: Draw the base of the solid and the slice.
Step 2: Find the equations of lines labled y_1 and y_2.



$y_2 = -2x + 4$

$y_1 = 4x + 4$

Step 3: Find the Area of the cross section.

The cross section is a semicircle so the area is $A = \frac{1}{2}\pi r^2$, where the radius r is half the length of the slice.

Length of the slice= $x_2 - x_1$ where x_1 is found by solving line y_1 for x. (Similar for x_2).

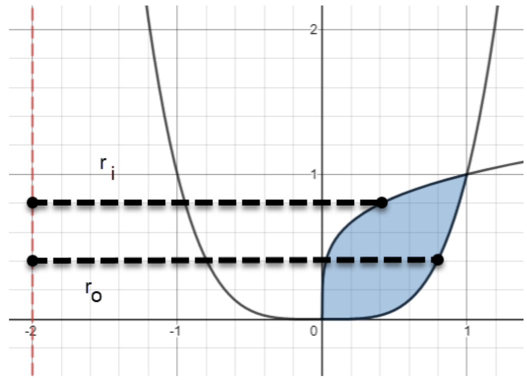
Length of the slice= $x_1 - x_2 = \frac{y-4}{-2} - \frac{y-4}{4} = \frac{12-3y}{4}$ so the radius is $r = \frac{1}{2}\left(\frac{12 - 3y}{4}\right) = \frac{12 - 3y}{8}$.

$$V = \int_{0}^{4} \frac{\pi}{2} \left(\frac{12 - 3y}{8}\right)^2 \, dy$$

3. Find the volume of the solid obtained by rotating the region bounded by the given curves around $x = -2$.

$y = x^4$
$x = y^4$

Step 1: Graph the functions.



The slice is not drawn, but a horizontal slice will be used and rotated about the vertical line $x = -2$. This means we will be using a dy integral.
The intersection points of the curves are $(0,0)$ and $(1,1)$.

Step 2: Find the formula for the inner and outer radius.

$r_o = \sqrt[4]{y} - (-2) = \sqrt[4]{y} + 2$

$r_i = y^4 - (-2) = y^4 + 2$

Step 3: Setup and evaluate the integral.

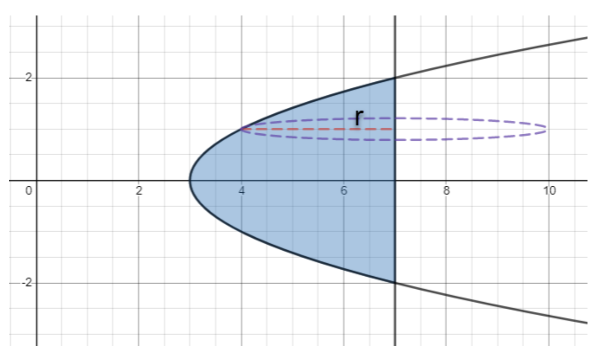
$$V = \int_0^1 \pi\left[\left(y^{0.25}+2\right)^2 - \left(y^4+2\right)^2\right]\,dy = \pi\int_0^1 y^{0.5} + 4y^{0.25} + 4 - \left(y^8 + 4y^4 + 4\right)\,dy$$

$$V = \pi\int_0^1 y^{0.5} + 4y^{0.25} - y^8 - 4y^4\,dy = \cdots = \frac{133\pi}{45}$$

4. Find the volume of the solid obtained by rotating the region bounded by the given curves around $x = 7$.

$x = y^2 + 3$
$x = 7$

Step 1: Graph the region.



Based on the slice that is shown, the integral will be a dy integral.
When $x = 7$ the $y-$values are $7 = y^2 + 3$ or $y^2 = 4$ or $y = \pm 2$.
Step 2: Find the radius.

The radius of the slice is $r = 7 - x = 7 - (y^2 + 3) = 4 - y^2$.
Step 3: set up the integral and evaluate.

$$V = \int_{-2}^{2} \pi(4-y^2)^2\,dy = 2\int_0^2 \pi(4-y^2)^2\,dy = 2\pi\int_0^2 16 - 8y^2 + y^4\,dy$$

$$V = 2\pi\left(16y - \frac{8y^3}{3} + \frac{y^5}{5}\right)\Bigg|_0^2 = 2\pi * \left(32 - \frac{64}{3} + \frac{32}{5}\right) = \frac{512\pi}{15}$$