

3) Find the Taylor series of $f(x) = xe^x$ about $a = -1$

$$f(x) = xe^x$$

$$f' = 1e^x + xe^x$$

$$f'' = e^x + (e^x + xe^x) \\ = 2e^x + xe^x$$

$$f''' = 2e^x + (e^x + xe^x) \\ = 3e^x + xe^x$$

notice that $f^{(n)}(x) = ne^x + xe^x$

$$\text{thus } f^{(n)}(-1) = ne^{-1} - 1e^{-1} = (n-1)e^{-1}$$

$$c_n = \frac{f^{(n)}(-1)}{n!} = \frac{(n-1)e^{-1}}{n!} = \frac{n-1}{e \cdot n!}$$

$$xe^x = \sum_{n=0}^{\infty} c_n (x - (-1))^n = \sum_{n=0}^{\infty} \frac{(n-1)}{e \cdot n!} (x+1)^n$$