3) Determine if the series is convergent or divergent. If convergent, then give the sum of the series.

A) $\sum_{n=1}^{\infty} (-5)^{n+2}4^{-n} = \sum_{n=1}^{\infty} \frac{(-5)^{n+2}}{4^n} = \sum_{n=1}^{\infty} \frac{(-5)^3}{4}\frac{(-5)^{n-1}}{4^{n-1}}$

Method #1

$= \sum_{n=1}^{\infty} \frac{-125}{4}\left(\frac{-5}{4}\right)^{n-1}$

$r = -\frac{5}{4}$ since $|r| > 1$ The series will diverge.

Method #2

$n=1$ $n=2$ $n=3$

$\sum_{n=1}^{\infty} (-5)^{n+2}(4)^{-n} = \frac{(-5)^3}{4} + \frac{(-5)^4}{4^2} + \frac{(-5)^5}{4^3} + \cdots\cdots$

$a = \frac{(-5)^3}{4}$ $ar = \frac{(-5)^4}{4^2} \implies r = \frac{-5}{4}$

since $|r| > 1$ The series will diverge.

B) $\sum_{i=3}^{\infty} 60\left(\frac{1}{2}\right)^i$ since $r=\frac{1}{2}$ and $|r|<1$

The series converges.

Method #1

$$\sum_{i=3}^{\infty} 60\left(\frac{1}{2}\right)^i = 60\left(\frac{1}{2}\right)^3 + 60\left(\frac{1}{2}\right)^4 + \cdots\cdots$$

now the geometric series has this sum

$$60 + 60\left(\frac{1}{2}\right) + 60\left(\frac{1}{2}\right)^2 + \underbrace{60\left(\frac{1}{2}\right)^3 + 60\left(\frac{1}{2}\right)^4 + \cdots}_{\sum_{i=3}^{\infty} 60\left(\frac{1}{2}\right)^i} = \frac{60}{1-\frac{1}{2}} = 120$$

Thus

$$\sum_{i=3}^{\infty} 60\left(\frac{1}{2}\right)^i = 120 - 60 - 60\left(\frac{1}{2}\right) - 60\left(\frac{1}{2}\right)^2$$

$$= 120 - 60 - 30 - 15$$

$$= 15$$

Method 2

$$\sum_{i=3}^{\infty} 60\left(\frac{1}{2}\right)^i = \underbrace{60\left(\frac{1}{2}\right)^3}_{a} + \underbrace{60\left(\frac{1}{2}\right)^4}_{ar} + \cdots\cdots$$

$a = 60 \cdot \left(\frac{1}{2}\right)^3$ $\quad r = \frac{1}{2}$

$$\text{Sum} = \frac{a}{1-r} = \frac{60\left(\frac{1}{2}\right)^3}{1-\frac{1}{2}} = \frac{60\left(\frac{1}{2}\right)^3}{\frac{1}{2}} = 60\left(\frac{1}{2}\right)^2 = \frac{60}{4}$$

$$= 15$$

C) $\sum_{n=2}^{\infty} 10 * \frac{2^{3n-1}}{5^{n+4}} = 10 \cdot \frac{2^5}{5^6} + 10 \cdot \frac{2^8}{5^7} + 10 \cdot \frac{2^{11}}{5^8} + \cdots$

$a = 10 \cdot \frac{2^5}{5^6}$ $r = \frac{2^3}{5} = \frac{8}{5}$ since $|r| > 1$

diverge

D) $\sum_{i=1}^{\infty} \left(e^{i^{-1}} - e^{(i+2)^{-1}}\right) = \sum_{i=1}^{n} \left(e^{\frac{1}{i}} - e^{\frac{1}{i+2}}\right)$

This is not a geometric series. It looks like a telescoping series, so lets find a formula for the partial sum.

$$S_n = \underbrace{\left(e^{1} - e^{\frac{1}{3}}\right)}_{i=1} + \underbrace{\left(e^{\frac{1}{2}} - e^{\frac{1}{4}}\right)}_{i=2} + \underbrace{\left(e^{\frac{1}{3}} - e^{\frac{1}{5}}\right)}_{i=3} + \underbrace{\left(e^{\frac{1}{4}} - e^{\frac{1}{6}}\right)}_{i=4}$$

$$+ \cdots + \underbrace{\left(e^{\frac{1}{n-3}} - e^{\frac{1}{n-1}}\right)}_{i=n-3} + \underbrace{\left(e^{\frac{1}{n-2}} - e^{\frac{1}{n}}\right)}_{i=n-2} + \underbrace{\left(e^{\frac{1}{n-1}} - e^{\frac{1}{n+1}}\right)}_{i=n-1} + \underbrace{\left(e^{\frac{1}{n}} - e^{\frac{1}{n+2}}\right)}_{i=n}$$

$$S_n = \left(e^{1} - \cancel{e^{\frac{1}{3}}}\right) + \left(e^{\frac{1}{2}} - \cancel{e^{\frac{1}{4}}}\right) + \left(\cancel{e^{\frac{1}{3}}} - \cancel{e^{\frac{1}{5}}}\right) + \left(\cancel{e^{\frac{1}{4}}} - \cancel{e^{\frac{1}{6}}}\right)$$

$$\cancel{+ \cdots +} \left(\cancel{e^{\frac{1}{n-3}}} - \cancel{e^{\frac{1}{n-1}}}\right) + \left(\cancel{e^{\frac{1}{n-2}}} - \cancel{e^{\frac{1}{n}}}\right) + \left(\cancel{e^{\frac{1}{n-1}}} - e^{\frac{1}{n+1}}\right) + \left(\cancel{e^{\frac{1}{n}}} - e^{\frac{1}{n+2}}\right)$$

$$S_n = e^{1} + e^{\frac{1}{2}} - e^{\frac{1}{n+1}} - e^{\frac{1}{n+2}}$$

$$\lim_{n\to\infty} S_n = e^{1} + e^{\frac{1}{2}} - e^{0} - e^{0}$$

$$= e + e^{1/2} - 2$$