

Challenge Problem:

1) Does this series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+4}}{6n^2-n-1}$$

Compare with $\sum \frac{\sqrt[3]{n^2}}{n^2} = \sum \frac{n^{2/3}}{n^2} = \sum \frac{1}{n^{4/3}}$

This is a p-series with $p = 4/3$ thus it converges

$$\lim_{n \rightarrow \infty} \frac{\frac{\sqrt[3]{n^2+4}}{6n^2-n-1}}{\frac{\sqrt[3]{n^2}}{n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2+4} \cdot n^2}{(6n^2-n-1) \sqrt[3]{n^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2+4}}{\sqrt[3]{n^2}} \cdot \frac{n^2}{6n^2-n-1}$$

$$= \lim_{n \rightarrow \infty} \underbrace{\sqrt[3]{\frac{n^2+4}{n^2}}}_{\text{goes to } \sqrt[3]{1}} \cdot \underbrace{\frac{n^2}{6n^2-n-1}}_{\text{goes to } \frac{1}{6}} = \sqrt[3]{1} \cdot \frac{1}{6} = \frac{1}{6}$$

used L'Hopital's to get this

By the limit comparison test both series will converge.