11.4 #4

note! the series has positive terms

by the LCT.

Known series $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty} \frac{2^n}{3^n}$ is geometric with $r = \frac{2}{3}$ so it converges.

$$\lim_{n\to\infty} \frac{\frac{1+2^n}{1+3^n}}{\frac{2^n}{3^n}} = \lim_{n\to\infty} \frac{3^n}{1+3^n} \cdot \frac{1+2^n}{2^n} = 1 \cdot 1 = 1$$

by LCT the series $\sum \frac{1+2^n}{1+3^n}$ will converge

L'Hopitals work.

$$\lim_{n\to\infty} \frac{3^n}{1+3^n} \overset{L'H}{=} \lim_{n\to\infty} \frac{3^n \cdot \ln(3)}{3^n \ln(3)} = 1$$

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$$\lim_{n\to\infty} \frac{1+2^n}{2^n} \overset{L'H}{=} \lim_{n\to\infty} \frac{2^n \ln(2)}{2^n \ln(2)} = 1$$