4) Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{\ln(n)(x-e)^n}{e^n}$$

Use the ratio test to find the radius of convergence.

$$\lim_{n\to\infty} \left| \frac{\ln(n+1)(x-e)^{n+1}}{e^{n+1}} \cdot \frac{e^n}{\ln(n)(x-e)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\ln(n+1) \cdot (x-e)}{\ln(n) \cdot e} \right| = \left| \frac{x-e}{e} \right|$$

note $\lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n)} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+1} = 1$

To have convergence we need this limit to be less than one.

$$\frac{|x-e|}{e} < 1$$

$$|x-e| < e \longrightarrow R = e$$

$$-e < x-e < e$$

$$0 < x < 2e$$

$$\sum_{n=0}^{\infty} \frac{\ln(n)(x-e)^n}{e^n}$$

$0 < x < 2e$

now test the endpoints of the interval.

$x = 2e$

$$\sum_{n=0} \frac{\ln(n)\,(2e-e)^n}{e^n} = \sum \frac{\ln(n)\,e^n}{e^n} = \sum \ln(n)$$

This series will diverge by the test for divergence.

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$x = 0$

$$\sum_{n=0} \frac{\ln(n)\,(0-e)^n}{e^n} = \sum_{n=0} (-1)^n \ln(n)$$

This series will diverge by the test for divergence.

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Conclusion:

$R = e$

$I = (0, 2e)$