

1) Find a power series for the function $f(x)$.

$$f(x) = \frac{5}{(1-5x)^2}$$

consider $g(x) = \frac{1}{1-5x}$

Then $g'(x) = \frac{5}{(1-5x)^2}$

Thus $f(x) = g'(x)$

Known Fact:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

with $|x| < 1$

$$g(x) = \frac{1}{1-5x} = \frac{1}{1-(5x)} = \sum_{n=0}^{\infty} (5x)^n \quad \text{where } |5x| < 1$$

ie $|x| < \frac{1}{5}$

$$= \sum_{n=0}^{\infty} 5^n x^n$$

$$f(x) = g'(x) = \sum_{n=1}^{\infty} 5^n \cdot n x^{n-1} \quad \text{with } |x| < \frac{1}{5}$$

While this information was not asked for, the radius of convergence is $R = \frac{1}{5}$

and the interval of convergence is $I = \left(-\frac{1}{5}, \frac{1}{5}\right)$