Math 325 | Exam 1 | Thursday, February 23, 2023

Printed Name: Key

Signature: ______________________

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

- Point values for each problem are as indicated.
- **To receive full credit for each problem, you must show all appropriate work, and your work must be presented in a clear, organized manner that is easy to follow.**
- If you need more space to work a problem, you may use the back of the exam. Please indicate where the problem is located.
- You may use up to two calculators on the exam.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Check the back of the page for problem.

1. (18 points) Given the amount function $A(t) = \dfrac{1000}{100 - t}$ for $0 \leq t < 100$.

   (a) Compute d_4 for this investment.

$$d_4 = \frac{A(4) - A(3)}{A(4)} = \frac{\frac{1000}{96} - \frac{1000}{97}}{\frac{1000}{96}} = \frac{1}{97} = 1.0309\%$$

   (b) Compute $a(20)$. $A(0) = K \Rightarrow K = \frac{1000}{100} = 10$

$$A(20) = K a(20)$$

$$\frac{1000}{80} = 10\, a(20) \rightarrow a(20) = \frac{1000}{80(10)} = \frac{10}{8} = 1.25$$

$$A' = \frac{1000}{(100-t)^2}$$

   (c) Compute δ_{10}

$$\delta_{10} = \frac{A'(10)}{A(10)} = \frac{\frac{1000}{90^2}}{\frac{1000}{90}} = \frac{1}{90} = 1.111\%$$

2. (10 points) Jack invest \$2,500 into an account now. How much will Jack have in 5 years if the account earns an annual simple discount rate of 7%?

$$2500\left(1 - (.07)5\right)^{-1} = \$3846.15$$

Check the back of the page for problem.

3. (12 points) Given that $i^{(6)} = 12\%$. compute the following. Give your answers as percentages with at least two decimal digits.

   (a) Give the equivalent constant force of interest.

$$\left(1+\frac{i^{(6)}}{6}\right)^6 = e^{\delta}$$

$$\delta = \ln\left[\left(1+\frac{.12}{6}\right)^6\right] = \ln\left[(1.02)^6\right] = 11.8815\%$$

   (b) Give $d^{(6)}$

$$\frac{d^{(6)}}{6} = \frac{\frac{i^{(6)}}{6}}{1+\frac{i^{(6)}}{6}} = \frac{.02}{1+.02} = .0196078$$

$$d^{(6)} = 6(.0196078) = .117647 \Rightarrow 11.7647\%$$

4. (8 points) Bob borrowed money from Sue on March 7, 2021, and will repay the debt on February 15, 2024. The loan is a simple interest loan with an annual simple interest rate. Compute the length of time(value of t) that will be used in the formulas for the following styles. IF you give a decimal answer, then provide 4 decimal places.

used BA calc.

   (a) ordinary simple interest: $t = \frac{1058}{360}$

30/360

dbd(3.0721, 2.1524, 360)

   (b) exact simple interest: $t = \frac{1075}{365}$

actual/365

dbd(3.0721, 2.1524, ~~365~~ ACT)

5. (13 points) An account is created that earns a force of interest of $\delta_t = 0.2 - 0.04t$ for $0 \leq t \leq 5$ and a constant force of interest of 3% thereafter. if \$800 is invested in this account at $t = 3$, find the accumulated value at time $t = 8$.

$$800\left(e^{\int_3^5 .2-.04t\,dt}\right)e^{.03(8-5)}$$

$$= 800\, e^{.08}\, e^{.09} = 800\, e^{.17}$$

$$= \$948.24$$

$$\int_3^5 .2 - .04t\,dt$$

$$= .2t - .02t^2 \Big|_3^5$$

$$= .2(5) - .02(5)^2 - [.2(3) - .02 \cdot 9$$

$$= .5 - .42$$

$$= .08$$

6. (13 points) Mason deposits \$700 into a bank account. Mason's account is credited with a nominal rate of interest of 8% convertible semiannually. At the same time Jacob deposits \$350 into a seperate account. Jacob's account is credited a constant force of interest δ. After 7.5 years the value of each account is the same. How much is in Jacob's account after 10 years?

Mason $i^{(2)} = 8\%$ Jacob δ = constant

PV = 700 PV = 350

$$700\left(1 + \frac{.08}{2}\right)^{2(7.5)} = 350\, e^{7.5\delta}$$

$$2(1+.04)^{15} = e^{7.5\delta}$$

$$\ln\left[2(1.04)^{15}\right] = 7.5\delta$$

$$\delta = \frac{\ln\left(2(1.04)^{15}\right)}{7.5} = 17.0861\%$$

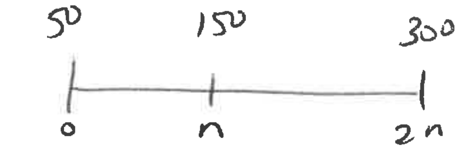
at 10 yrs

$$350\, e^{10\delta}$$

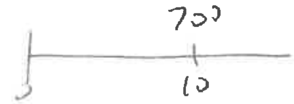
$$= \$1932.45$$

Check the back of the page for problem.

7. (13 points) Joe can recieve one of the following two payment streams:



Payment stream 1:
\$50 at time 0, \$150 at time n, and \$300 at time $2n$



Payment stream 2:
\$700 at time 10

At an annual effective interest rate of i, the present value of the two streams are equal. Given $v^n = 0.73$, determine i.

$$50 + 150v^n + 300v^{2n} = 700v^{10}$$

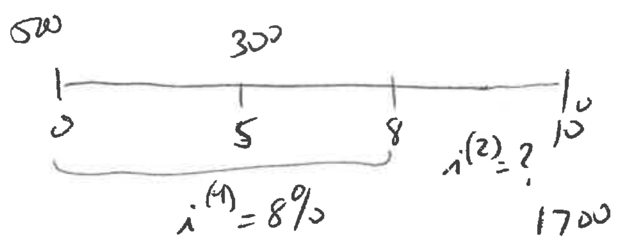
$$v^{10} = \frac{50 + 150(.73) + 300(.73)^2}{700} = .45624$$

$$\frac{1}{(1+i)^{10}} = .45624$$

$$(1+i)^{10} = (.45624)^{-1}$$

$$i = (.45624)^{-1/10} - 1$$
$$= 8.163415\%$$

8. (13 points) \$500 is deposited into an account now and another \$300 is deposited into the account at time t= 5 years. The account earns interest at a rate of $i^{(4)} = 8\%$ from time t=0 to time t=8(years). At time t=8 the rate earned by the acocunt changes. What nominal rate of interest compounded semiannually from t=8 to t= 10 would produce an accumulation of \$1700 at t=10?



$$1700 = \left[500\left(1+\frac{i^{(4)}}{4}\right)^{20} + 300\right]\left(1+\frac{i^{(4)}}{4}\right)^{12}\left(1+\frac{i^{(2)}}{2}\right)^4$$

$$1700 = \left[500(1.02)^{20} + 300\right](1.02)^{12}\left(1+\frac{i^{(2)}}{2}\right)^4$$

$$1700 = 1322.74\left(1+\frac{i^{(2)}}{2}\right)^4$$

$$i^{(2)} = 2\left[\left(\frac{1700}{1322.74}\right)^{1/4} - 1\right] = 12.947\%$$

Check the back of the page for problem.