

**HW #1. Math 611-600. 5 pts/problem.
Fall 2017. Due 09/12/2017**

General remarks:

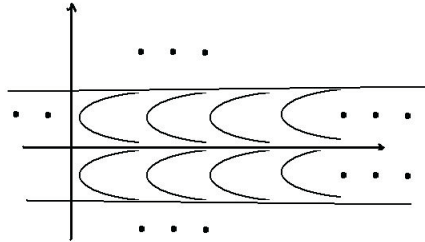
- **Write solutions neatly and not in microscopic letters.** Avoid many crossing outs, etc. This is not only for me to read easier, but for you to be sure that what you write is correct. I will not grade solutions that are messily written and unreadable due to a tiny letter size.
- **Avoid words “trivial ..., easy to see ..., obvious ...” and other their synonyms.** “Easy to see” or “trivial” should mean that you can show this in a second, so just do it :-). By the way, **errors are usually appearing in “obvious” places.**
- **Be careful with parentheses and other such standard rules;** they can cause mistakes if not followed properly.
- **Make sure you understand your logic completely.** If you don't, I won't. :-)

Problems:

1. Solve by successive Picard iterations the initial value problem

$$\begin{cases} \frac{dx}{dt} = x \\ x(0) = 1 \end{cases} .$$

2. Show that the fields arising from turning non-autonomous systems into autonomous ones, are always non-singular at all points.
3. Deduce the existence and uniqueness theorem from the theorem on rectifying a vector field near a non-singular point.
4. Show that the vector field rectification theorem implies the dependence on parameters theorem.
5. Does a **global** rectifying theorem hold? If yes, why. If not, give an example of not globally rectifiable non-singular vector field. **HINT:** Look at the following phase curves:



6. Provide an example of an initial value problem for an ODE that fails to have any solution.
7. Provide an example of a “nice” boundary value problem for an ODE that fails to have any solution.
8. Review Appendix A and solve exercises # # 1, 3, 5, and 4 on p. 12 of the textbook.