

## Math308, Section 200 Spring 2011

### Contents of the Final Exam

- **Use of computers (no prearranged worksheets) and the Laplace transforms table is allowed.**
  - **You will need to show work, mere answers will not be accepted.**
  - **Calculators will not be allowed.**
  - **The test will contain some problems involving Maple.**
  - **Besides knowing how to compute solutions, you need to know main theorems and understand the main notions.**
1. The notions of a differential equation and its solution. Sections 1.1, 1.2.
  2. Classification: ordinary diff. equations (ODEs) vs partial differential equations (PDEs), linear vs nonlinear, order of an equation. Section 1.4.
  3. Initial conditions, initial value problem (IVP).
  4. Existence and uniqueness theorem for 1st order ODEs. Section 2.4.
  5. Direction fields and integral curves. Section 1.1.
  6. Identifying and solving the main types of 1st order equations: separable, linear, exact,
  7. homogeneous, and Bernoulli. Sections 2.1--2.6.
  8. Solving ODEs and IVPs using Maple (both symbolic and numerical solutions).
  9. Approximate solutions of 1st order ODEs using Euler and improved Euler methods (Sect. 1.3, 2.7, 2.8).
  10. Systems of two first order ODEs. Matrix form in the linear case. Sections 3.1, 3.2.
  11. Matrix algebra. Eigenvalues and eigenvectors of matrices. Characteristic equation. Sect. 3.3, A.1-A.4.
  12. Superposition principle, fundamental set of solutions, and the structure of the general solution for homogeneous and inhomogeneous linear systems of two 1<sup>st</sup> order ODEs.
  13. Finding the general solution in the cases of real distinct, complex, and real repeated roots. Classification of phase portraits. Instability, stability, asymptotic stability. Sect. 3.3 – 3.6.
  14. Second order linear differential equations (homogeneous and inhomogeneous). Section 4.1 – 4.3.
  15. Linear independence and dependence of a pair of functions. Wronskian and its use for testing independence of solutions of a homogeneous linear second order equation. Section 4.2 – 4.3.
  16. Structure of the general solution of a homogeneous second order linear equation. Solving homogeneous linear second order equations with constant coefficients. The characteristic equation and finding the general solution of the ODE (3 cases). Solving IVPs for such equations. Sections 4.2 - 4.4.
  17. Solving inhomogeneous linear second order equations with constant coefficients and the right hand side of special types. Section 4. 6.
  18. Free mechanical vibrations of a spring-mass system and electric oscillations of an RIC circuit. Damping and its role (undamped, underdamped, critically damped, and overdamped systems and their properties). Section 4.5.

19. Forced vibrations. Gain factor. Resonance. Transient and steady state parts of a solution. Sections 4.7.
20. Laplace transform, its properties, and its computation by the definition and using the table. Sections 5.1 – 5.2.
21. Inverting the Laplace transform. Section 5.3.
22. Solving IVPs for linear constant coefficient ODEs using Laplace transform. Section 5.4.

### **Extra credit**

1. Variation of parameters in 2<sup>nd</sup> order linear equations Section 4.8
2. Nonlinear systems of ODEs and stability. Competing species and predator-prey equations. Sections 7.1 -7.4.
3. Using Maple for numerical solving ODEs.