

**Differential Equations, Math. 308–200. Spring 2011.**

**Instructor P. Kuchment.**

**Assignment A3. Due February 10th.**

15 points (1 point for each of the problems ## 1–5, 2 points for each of the problems ## 6–10). Show all work.

In the following IVPs determine what the existence and uniqueness theorems can say about existence, domain of existence, and uniqueness of solution.

1.

$$\begin{cases} \ln |t - 1| y' - 2ty = t^2 \\ y|_{t=2} = 3 \end{cases}$$

2. The same equation with  $y|_{t=1} = 3$

3.

$$\begin{cases} \frac{dv}{ds} + \sqrt[3]{v} = s \\ v|_{s=2} = 0 \end{cases}$$

4. The same equation with  $v|_{s=2} = 1$

5.

$$\begin{cases} y' = e^{-\frac{1}{x^2}} y + \sin x \\ u|_{x=0} = 1 \end{cases}$$

Determine the type (**write it down**) and solve the following four equations or IVPs.

6.

$$\begin{cases} (1 + y^2 \sin 2x) dx = 2y \cos^2 x dy \\ y|_{x=0} = 1 \end{cases}$$

7.

$$xy' = \sqrt{x^2 - y^2} + y$$

8.

$$2x^2 y' = y^3 + xy$$

9.

$$e^{-y} = y'(2y + xe^{-y})$$

10. Consider the equation

$$(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0$$

- (a) Show that the equation is not exact
- (b) Multiply the equation by  $x^n y^m$  and determine values of  $n$  and  $m$  that make the resulting equation exact.
- (c) Use the solution of the resulting exact equation to solve the original equation.