**Differential Equations, Math. 308–200. Spring 2011.**
**Instructor P. Kuchment.**
**Assignment A5. Due March 10th.**
18 points (1 point for each problem). Show all work.

1. Find the matrix of the following system and write the system in the vector form:
$$\begin{cases} 3x_1 - 2x_2 = 5 \\ 2x_2 + x_1 = 7 \end{cases}$$

2. Compute $A\mathbf{x}$, where $A = \begin{pmatrix} 2 & -1 \\ \pi & 0 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

3. Compute $AB$ and $BA$, where $A = \begin{pmatrix} 2 & -1 \\ \pi & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.

4. The matrix $A = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$ creates a linear mapping (transformation) of the plane into itself. What are the images of the standard basis vectors $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

5. For the same basis vectors, find a matrix $A$ such that $Ae_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $Ae_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. Is such a matrix unique?

6. Find the transpose $A^t$ to the matrix $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

7. Compute $\det A$ and $\mathrm{tr}A$ for the matrix in the previous exercise.

8. What is the area of the parallelogram that has vectors $a = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ as its sides?

9. A matrix $A$ sends the standard basis vectors $e_1,\ e_2$ (see Problem 4) into two parallel (collinear) vectors. What is the determinant of $A$?

10. What does it mean that a matrix is non-singular? Write examples of a non-singular and of a singular matrices (with non-zero entries).

11. Find the inverse for the matrix $A$ in Problem 4.

12. Use the result of the previous problem and matrix algebra to solve the system

$$\begin{cases} 3x_1 + x_2 = 5 \\ x_1 + x_2 = 7 \end{cases}$$

13. Let $\lambda_1, \lambda_2$ be the eigenvalues of a matrix $A$. Prove that $\lambda_1 + \lambda_2 = \mathrm{tr}A$, $\lambda_1 \times \lambda_2 = \det A$.

14. Can a matrix $A$ with real entries have eigenvalues $\lambda_1 = 2 + i, \lambda_2 = 3$?

15. Construct an example of a matrix that has eigenvalues $\lambda_1 = 2, \lambda_2 = 3$.

Find eigenvalues and corresponding eigenvectors of the following matrices:

16. $A = \begin{pmatrix} 5 & 3 \\ -4 & -2 \end{pmatrix}$

17. $A = \begin{pmatrix} 4 & 5 \\ -5 & -6 \end{pmatrix}$

18. $A = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$