

**Math308-200. Spring 2011**  
**Home Assignment #7. Due April 19th.**  
**2 points per a problem. No partial credits.**

**In problems ## 1–4 find the general solution of the given differential equation:**

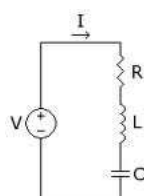
1.  $y'' - y' - 2y = 0$
2.  $4y'' - 4y' + y = 0$
3.  $y'' - 4y' + 7y = 0$
4.  $\frac{d^2u}{dx^2} + 6u = 0$

**In problems ## 5–7 solve the initial value problem:**

5.  $y'' + y' = 0, y(0) = 2, y'(0) = 1$
6.  $y'' + 2y' + 17y = 0, y(0) = 1, y'(0) = -1$
7.  $y'' - 4y' + 4y = 0, y(1) = 1, y'(1) = 1$   $y = e^{2t-2}(2 - t)$
8. To see the effect of changing the parameter  $\gamma$  in the initial value problem  $y'' + \gamma y' + 4y = 0, y(0) = 1, y'(0) = 0$ , solve the problem for  $\gamma = 5, 4$ , and  $2$ , and sketch the solutions.
9. A vibrating spring without damping is modeled by an equation similar to the one in the previous problem (see Section 4.5 in the textbook), with  $\gamma = 0$ . If  $m = 10kg$ ,  $k = 250kg/sec^2$ ,  $y(0) = 0.3m$ , and  $y'(0) = -0.1m/sec$ , solve the vibrating spring differential equation to find the equation of motion of the undamped spring.
10. A vibrating spring with damping has parameters  $m = 10kg$ ,  $\gamma = 60kg/sec$  (damping),  $k = 250kg/sec^2$ ,  $y(0) = 0.3m$ , and  $y'(0) = -0.1m/sec$ . Solve the corresponding differential equation to find the equation of motion of the damped spring. Compare the results of these two problems. In particular, how does the presence of damping affect the frequency of oscillation?

**Extra credit, 3 points per a problem (no partial credits)**

11. Try to generalize the approach used for the 2nd order equations to find the general solution of the 3rd order equation  $y''' + y'' - 6y' + 4y = 0$ .
12. Electric oscillations in the RLC series circuit (see the Fig. below)



can be described by the equation

$$L \frac{dI}{dt} + RI + \frac{Q}{C} = V(t),$$

where  $t$  is time,  $I(t)$  is the current in *amperes*,  $Q(t)$  is the charge on the capacitor in *coulombs*, and  $L, R, C$  are correspondingly inductance in *henrys*, resistance in *ohms*, and capacitance in *farads*.  $V(t)$  is the electromotive force in *volts*. There is the relation  $dQ/dt = I$  between the charge and the current. Find the current at time  $t$ , if the charge on the capacitor is initially zero, the initial current is zero,  $L = 10\text{henrys}$ ,  $R = 20\text{ohms}$ ,  $C = (6260)^{-1}\text{farads}$ , and  $V(t) = 100\text{volts}$ . (**Hint:** find an equation for the current alone.)