The purpose of this note is to make a minor correction to Theorem 1.6 of the paper “Nonvanishing of Hecke $L$-functions and Bloch–Kato $p$–Selmer groups.” Math. Res. Lett. 26 (2019), no. 4, 1145–1177. We would like to thank Neil Dummigan for alerting us to the need for this correction.

On page 1151, the paragraph beginning after line 11 should be replaced with the following text:

Let $p$ be a prime which is coprime to $d$ and $2k − 1$. Define the set

$$S_{d,k,p} := \{ K : K \in S_{d,k} and p \text{ splits in } K \}.$$ 

For $K = \mathbb{Q}(\sqrt{-D}) \in S_{d,k,p}$, we denote the number of canonical Hecke characters in the family $\Psi_{d,k}(D)$ whose associated Bloch-Kato $p$-Selmer group is finite by

$$FS_{d,k,p}(D) := \# \{ \psi \in \Psi_{d,k}(D) : \# \text{Sel}_p(A(\psi)/K) < \infty \}.$$ 

Then, for fixed $\varepsilon > 0$ we define the following subset of $S_{d,k,p}(X)$:

$$S_{FS}^{d,k,p,\varepsilon}(X) := \{ K \in S_{d,k,p}(X) : FS_{d,k,p}(D) \gg \varepsilon D^{\frac{1}{2(2k-1)} - \varepsilon} \}.$$ 

**Theorem 1.6.** We have

$$\#S_{FS}^{d,k,p,\varepsilon}(X) = \delta(d,k,p)X + O_{d,k,p}(X^{1/2})$$

where

$$\delta(d,k,p) = \frac{1}{2} \left( \frac{1}{1 + p^{-1}} \right) \delta(d,k)$$

and

$$\frac{\#S_{FS}^{d,k,p,\varepsilon}(X)}{\#S_{d,k,p}(X)} = 1 + O_{d,k,p}(X^{-1/2})$$

as $X \to \infty$. In particular, 100% of imaginary quadratic fields $K \in S_{d,k,p}$ satisfy the bound

$$FS_{d,k,p}(D) \gg \varepsilon D^{\frac{1}{2(2k-1)} - \varepsilon}.$$ 

**Proof.** One can use the condition (1.1) to show that a canonical Hecke character $\psi \in \Psi_{d,k}(D)$ satisfies $\overline{\psi}(\mathfrak{a}) = \psi(\overline{\mathfrak{a}})$ for integral ideals $\mathfrak{a}$ of $K$ which are prime to $d\sqrt{-DO_K}$ (see [28]). Since the conductor of $\psi$ is $f = d\sqrt{-DO_K}$, it follows that $\overline{\psi}(\mathfrak{a}) = \psi(\overline{\mathfrak{a}})$ for all integral ideals $\mathfrak{a}$ of $K$, and thus $L(\overline{\psi}, s) = L(\psi, s)$. Now, by a minor modification of the proofs of Theorem 1.2 and Corollary 1.3 which incorporates the additional condition that the prime $p$ split in all imaginary quadratic fields being counted, we get the asymptotic formulas

$$\#S_{NV}^{d,k,p,\varepsilon}(X) = \delta(d,k,p)X + O_{d,k,p}(X^{1/2}) \quad (0.1)$$

where

$$\delta(d,k,p) = \frac{1}{2} \left( \frac{1}{1 + p^{-1}} \right) \delta(d,k)$$
and
\[
\frac{\#S_{d,k,p}^{NV}(X)}{\#S_{d,k,p}(X)} = 1 + O_{d,k,p}(X^{-1/2}) \tag{0.2}
\]
as \(X \to \infty\). The result now follows immediately from (0.1), (0.2), and Theorem 1.5. □