Section 1.2: Break-Even Analysis and Market Equilibrium

A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomenon. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

**Linear Demand and Supply Curves**

**Example 1:** For a particular commodity, it is found that 20 units will be supplied at a unit price of $9 whereas 10 units will be supplied at a unit price of $7. For this commodity, it is found that the consumers are willing to consume 10 units at a unit price of $23 but are only willing to consume 2 units at a unit price of $24.60. Assuming that both supply and demand functions are linear, find the

a) supply equation

\[
\text{Two points: (20, 9) and (10, 7)}
\]

\[
\text{Slope} \quad \frac{9 - 7}{20 - 10} = \frac{2}{10} = .2
\]

\[
\text{Point - Slope:} \quad p - 7 = .2(x - 10) \Rightarrow p = .2x + 5
\]

b) demand equation

\[
\text{Two points: (10, 23) and (2, 24.60)}
\]

\[
\text{Slope:} \quad \frac{24.60 - 23}{2 - 10} = - .2
\]

\[
\text{Point - Slope:} \quad p - 23 = - .2(x - 10) \Rightarrow p = - .2x + 23
\]

**Market Equilibrium:** The point at which the consumer and supplier agree upon (i.e. the point of intersection of the supply and demand curves)

**Example 2:** Find and interpret the market equilibrium for the commodity in Example 1.

\[
\text{Supply} = \text{demand} \quad \frac{.4x = 20}{.4} \Rightarrow x = 50 \quad \Rightarrow \quad p = .2(50) + 5 = $15
\]

**Equilibrium point** \((50, 15)\)
Cost, Revenue, and Profit Functions

**Cost Function** (cost of manufacturing \( x \) units of a product)

\[
C(x) = cx + F
\]

where \( c \) is the cost per unit and \( F \) is the fixed cost.

**Revenue Function** (revenue realized from selling \( x \) units of the product)

\[
R(x) = sx, \text{ where } s \text{ is the sell price}
\]

**Profit Function** (profit realized from making and selling \( x \) units of the product)

\[
P(x) = R(x) - C(x) = (s - c)x - F
\]

**Example 3:** A manufacturer of garbage disposals, has a monthly fixed cost of $14,000 and a production cost of $5 for each garbage disposal manufactured. The units sell for $75 each.

a) What is the cost function?

\[
C(x) = 75x + 14000
\]

b) What is the revenue function?

\[
R(x) = 75x
\]

c) What is the profit function?

\[
P(x) = 70x - 14000
\]

**Break-Even Point:** The point at which the company suffers neither a loss or gain. (i.e. the point of intersection of the revenue and cost functions: \( R(x) = C(x) \) or \( P(x) = 0 \))

**Example 4:** Find (and interpret) the break-even point for the garbage disposals in Example 3.

\[
egin{align*}
75x &= 5x + 14000 \\
-5x &= -5x \\
70x &= 14000 \\
&= 200
\end{align*}
\]

\( x \approx 187 \)

If you make and sell 187 garbage disposals then you have no profit.
Quadratic Revenue and Profit Functions

Example 5: It is found that the consumers of a particular toaster will demand 64 toaster ovens when the unit price is $35 whereas they will demand 448 toaster ovens when the unit price is $5. Assuming that the demand function is linear and the selling price is determined by the demand function,

a) Find the demand equation.

Two points: \((64, 35), (448, 5)\)

\[
\text{Slope} = \frac{35 - 5}{64 - 448} = \frac{-5}{64}
\]

Point-Slope

\[p - 35 = \frac{-5}{64}(x - 64)\]

\[p = \frac{-5}{64}x + 40\]

b) Find the revenue function.

c) Find the number of items sold that will give the maximum revenue. What is the maximum revenue?

Revenue: \(R(x) = p(x) \cdot x = \left(\frac{-5}{64}x + 40\right)x\)

\[= \frac{-5}{64}x^2 + 40x\]

Maximum occurs at vertex. Vector: \((h, k)\). “\(h\) is items to maximize and \(k\) is max revenue. \(h = \frac{-b}{2a} = \frac{-40}{2 \left(\frac{-5}{64}\right)} = 256\)

\[k = R(256) = \$5120\]

d) If the company has a fixed cost of $1,000 and a variable cost of $15 per toaster, find the company’s linear cost function.

\[F = 1000\]

\[C = 15\]

\[C(x) = 15x + 1000\]

e) What is the company’s maximum profit?

\[P(x) = R(x) - C(x) = \frac{-5}{64}x^2 + 25x - 1000\]

Max at vertex \((h, k)\). \(h = P(160) = \$1000\)

\[k = \frac{-25}{2 \left(\frac{-5}{64}\right)} = 160\]

f) How many toasters should be sold for the company to break even?

\[P(x) = 0 \Rightarrow \frac{-5}{64}x^2 + 25x - 1000 = 0\]

Quadratic Formula:

\[X = \frac{-25 \pm \sqrt{25^2 - 4\left(\frac{-5}{64}\right)(-1000)}}{2\left(\frac{-5}{64}\right)} \Rightarrow X \approx 47 \text{ or } 273\]