Section 5.1 Compound Interest

Simple Interest Formulas:

Interest: \( I = Prt \)

Accumulated amount: \( A = P(1 + rt) \)

Here \( P \) is the principal (money you start out with), \( r \) is the interest rate (as a decimal), and \( t \) is the time (in years).

1. Find the accumulated amount at the end of 9 months on a $1800 bank deposit paying simple interest at a rate of 9%/year. (Round answer to the nearest cent.)

\[
\begin{align*}
P &= 1800, \ r &= 0.09, \ t &= \frac{3}{12} \\
A &= 1800 \left(1 + (0.09 \times \frac{3}{12})\right) = \$1921.50
\end{align*}
\]

2. A bank deposit paying simple interest at the rate of 6%/year grew to a sum of $1300 in 8 months. Find the principal. (Round answer to the nearest cent.)

\[
\begin{align*}
A &= 1300, \ r &= 0.06, \ t &= \frac{8}{12} \\
1300 &= P(1 + (0.06 \times \frac{8}{12})) \\
\frac{1300 - P}{0.04} &= P \\
\Rightarrow P &= \$1250
\end{align*}
\]

3. Determine the simple interest rate at which $2400 will grow to $2495 in 5 months. (Round answer to two decimal places.)

\[
\begin{align*}
P &= 2400, \ A &= 2495, \ t &= \frac{5}{12} \\
\frac{495}{2400} &= \frac{2495}{2400} \left(1 + (\frac{5}{12})r\right) \\
\frac{495}{2400} - 1 &= \frac{(\frac{5}{12})r}{(\frac{5}{12})} \\
r &= \frac{(\frac{495}{2400} - 1)}{(\frac{5}{12})} \\
r &= 0.095 \\
\Rightarrow r &= 9.5\%/year
\end{align*}
\]
Compounded Interest Formulas: Accumulated Amount

\[ A = P(1 + i)^n \]

where \( i = \frac{r}{m} \), \( n = mt \), and 

\( A \) = Accumulated amount at the end of \( n \) conversion periods.  
\( P \) = Principal.  
\( r \) = Nominal interest rate per year.  
\( m \) = Number of conversion periods per year.  
\( t \) = Term (number of years)

Calculator Functions

**TVM Solver:** We can use the TVM Solver on our calculator to solve problems involving compound interest. To access the Finance Menu, you need to press [APPS] [1] and then [1] again. (Please note that if you have a plain TI-83, you need to press [2ND] [x^{-1}] to access the Finance Menu). Below we define the inputs on the TVM Solver:

\( N = mt \) =the total number of compounding periods 
\( I\% = \) interest rate (as a percentage) 
\( PV = \) present value (principal amount). Entered as a negative number if invested, a positive number if borrowed. 
\( PMT = \) payment amount 
\( FV = \) future value (accumulated amount) 
\( P/Y = C/Y = m \) =the number of compounding periods per year.

Move the cursor to the value you are solving for and hit ALPHA and then ENTER.

4. Find the present value of $40,000 due in 4 years at the given rate of interest. (Round answer to the nearest cent.)

- 10%/year compounded daily.
  
  \[ \begin{align*}
  N &= (365 \times 4) \\
  I\% &= 10 \\
  PV &= -26814.27 \\
  PMT &= 0 \\
  FV &= 40000 \\
  P/Y = C/Y &= 365
  \end{align*} \]

\[ \boxed{\$ 26,814.27} \]
5. A young man is the beneficiary of a trust fund established for him 16 years ago at his birth. If the original amount placed in trust was $20,000, how much will he receive if the money has earned interest at the rate of 9%/year compounded quarterly? (Round answer to the nearest cent.)

\[ N = (4 \times 16) \]
\[ I\% = 9 \]
\[ PV = \$20,000 \]
\[ PMT = 0 \]
\[ FV = \$83077.28 \]
\[ P/Y = C/Y = 4 \]

6. Five and a half years ago, Chris invested $10,000 in a retirement fund that grew at the rate of 10.82%/year compounded quarterly. What is his account worth today? (Round answer to the nearest cent.)

\[ N = (5.5 \times 4) \]
\[ I\% = 10.82 \]
\[ PV = \$10000 \]
\[ PMT = 0 \]
\[ FV = \$17,489.33 \]
\[ P/Y = C/Y = 4 \]

7. Kim invested a sum of money 7 years ago in a savings account that has since paid interest at the rate of 8.5%/year compounded monthly. Her investment is now worth $36,184.65. How much did she originally invest? (Round answer to the nearest cent.)

\[ N = (7 \times 12) \]
\[ I\% = 8.5 \]
\[ PV = \$20000\]
\[ PMT = 0 \]
\[ FV = 36184.65 \]
\[ P/Y = C/Y = 12 \]
8. Your rich uncle has just given you a high school graduation present of $1,400,000. The present, however, is in the form of an 18-year bond with an annual interest rate of 4.7% compounded annually. The bond says that it will be worth $1,400,000 in 18 years. What is this gift worth at the present time? (Round answer to the nearest cent.)

\[ N = (1 + r)^t \]
\[ I\% = 4.7 \]
\[ PV = ? \]
\[ PMT = 0 \]
\[ FV = 1,400,000 \]
\[ P/Y = C/Y = 1 \]

Effective Rate of Interest Formula:

\[ r_{eff} = \left( 1 + \frac{r}{m} \right)^m - 1 \]

Calculator Steps:

Press [APPΣ] 1, scroll down to [Eff] and hit ENTER.

The format is [Eff(annual interest rate as a percentage, the number of compounding periods per year)]

9. Find the effective rate of interest corresponding to a nominal rate of 11.5%/year compounded in the following ways. (Round answers to two decimal places.)

(a) compounded annually

\[ Eff(11.5, 1) = 11.5\% \]

(b) compounded semiannually

\[ Eff(11.5, 2) = 11.83\% \]

(c) compounded quarterly

\[ Eff(11.5, 4) = 12.01\% \]
(d) compounded monthly

\[ \text{Eff}(11.5, 12) \approx 12.13\% \]

Continuous Compound Interest Formula: Accumulated Amount

\[ A = Pe^{rt} \]

10. Find the accumulated amount after 2 years if $4200 is invested at 3%/year compounded continuously. (Round answer to the nearest cent.)

\[ A = 4200e^{(0.03)(2)} = $4459.71 \]
Section 5.2 Annuities

**Future Value of an Annuity** The future value $S$ of an annuity of $n$ payments of $R$ dollars each, paid at the end of each investment period into an account that earns interest at the rate of $i$ per period, is

$$S = R \left[ \frac{(1 + i)^n - 1}{i} \right]$$

**Note:** We will continue to use the TVM solver only this time the “PMT” entry will not be zero for most of the following problems. Also, we will make the value negative.

1. Robin, who is self-employed, contributes $5000/year into an account. How much will he have in the account after 25 years if the account earns interest at the rate of 8.5%/year compounded yearly? (Round answer to the nearest cent.)

   - **N** = (11x25)
   - **I%** = 8.5
   - **PV** = 0
   - **PMT** = -5000
   - **FV** = $393,338.96
   - **P/Y** = C/Y = 1

   $393,338.96$

2. The Pirerras are planning to go to Europe 2 years from now and have agreed to set aside $140/month for their trip. If they deposit this money at the end of each month into a savings account paying interest at the rate of 6.5%/year compounded monthly, how much money will be in their travel fund at the end of the second year? (Round answer to the nearest cent.)

   - **N** = (2x12)
   - **I%** = 6.5
   - **PV** = 0
   - **PMT** = -140
   - **FV** = $3,577.84
   - **P/Y** = C/Y = 12

   $3,577.84$
3. Lauren plans to deposit $4000 into a bank account at the beginning of next month and $250/month into the same account at the end of that month and at the end of each subsequent month for the next 5 years. If her bank pays interest at a rate of 5%/year compounded monthly, how much will Lauren have in her account at the end of 5 years? (Assume she makes no withdrawals during the 5-year period. Round answer to the nearest cent.)

\[
\begin{align*}
N &= (5 \times 12) \\
I\% &= 5 \\
PV &= -4000 \\
PMT &= -250 \\
FV &= 22,134.96 \\
P/Y &= C/Y = 12
\end{align*}
\]

Present Value of an Annuity The present value \( P \) of an annuity consisting of \( n \) payments of \( R \) dollars each, paid at the end of each investment period into an account that earns interest at the rate of \( i \) per period, is

\[
P = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]
\]

Note: The future value \( S \) does not appear in the above formula. This means that when using the TVM solver that the entry “FV” will be zero.

4. Find the amount needed to deposit into an account today that will yield pension payments of $35,000 at the end of each of the next 29 years if the account earns interest at a rate of 5.9%/yr compounded annually. (Round answer to the nearest cent.)

\[
\begin{align*}
N &= (1 \times 29) \\
I\% &= 5.9 \\
PV &= 480,699.44 \\
PMT &= -35,000 \\
FV &= 0 \\
P/Y &= C/Y = 1
\end{align*}
\]
5. A local moving service recently purchased a van by securing a loan with semiannual payments of $2900 per semiannual period for 6 years at 12% per year compounded semiannually. What was the purchase price of this van? (Round answer to the nearest cent.)

\[ N = (6 \times 2) = 12 \]
\[ I\% = 12 \]
\[ PV = ? \]
\[ PMT = -2900 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 2 \]

Price of car
$24,313.15

6. Lupe made a down payment of $2200 toward the purchase of a new car. To pay the balance of the purchase price, she has secured a loan from her bank at the rate of 13%/year compounded monthly. Under the terms of her finance agreement she is required to make payments of $240/month for 48 months. What is the cash price of the car? (Round your answer to the nearest cent.)

\[ N = 48 \]
\[ I\% = 13 \]
\[ PV = ? \]
\[ PMT = -240 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

Price of car = PV + Down Payment

= 8946.05 + 2200

= $11,146.05
Section 5.3 Amortization and Sinking Funds

Sinking Fund Payments With these problems we will solve for the payment $PMT$ required to obtained a given accumulated amount of money $FV$. In this case $PV$ will be zero.

1. Andrea, a self-employed individual, wishes to accumulate a retirement fund of $450,000. How much should she deposit each month into her retirement account, which pays interest at a rate of 3.5%/year compounded monthly, to reach her goal upon retirement 40 years from now? (Round answer to the nearest cent.)

   $N = (12 \times 40)$
   $I\% = 3.5$
   $PV = 0$
   $PMT = \_430.76$
   $FV = 450000$
   $P/Y = C/Y = 12$

   Interest Earned Interest earned on a sinking fund where $N$ payments of $PMT$ dollars have accumulated to $FV$ dollars is given by,

   $FV - N \times PMT$

2. A corporation creates a sinking fund in order to have $540,000 to replace some machinery in 12 years. How much should be placed in this account at the end of each quarter if the annual interest rate is 4.1% compounded quarterly? (Round answers to the nearest cent.)

   $N = (4 \times 12)$
   $I\% = 4.1$
   $PV = 0$
   $PMT = \_8764.94$
   $FV = 540000$
   $P/Y = C/Y = 4$

   $8,764.94$

Fall 2017, © Maya Johnson
Determine the value of the fund after 6 years.

\[ \text{Solve for } FV \text{ and change } N \text{ to } (4 \times 6) \]

\[ FV = 1237123 \]

How much interest would they earn over the life of the account?

\[ FV_{12} - N \times PMT = 250000 - (4 \times 12)(5764.94) = 119282.88 \]

**Amortization Payments** With these problems we will solve for the payment \( PMT \) required to amortize a loan of \( PV \) dollars. Amortization means we pay-off the loan so that \( FV \) is zero.

3. Carl is the beneficiary of a $28,000 trust fund set up for him by his grandparents. Under the terms of the trust, he is to receive equal installments from this fund at the end of each year over a 5-year period. If the fund earns interest at the rate of 10%/year compounded annually, what amount will he receive each year? Assume that the balance in the fund is zero after the last installment is received. (Round answer to the nearest cent.)

\[ \begin{align*} N &= (1 \times 5) \\ I\% &= 10 \\ PV &= 28000 \\ PMT &= 7786.33 \\ FV &= 0 \\ P/Y &= C/Y = 1 \end{align*} \]

**Interest Paid** Interest paid on an amortization of a loan of \( PV \) dollars where \( N \) payments of \( PMT \) dollars have been made is given by,

\[ N \times PMT - PV \]
4. Find the monthly payment needed to amortize a typical $140,000 mortgage loan amortized over 30 years at an annual interest rate of 6.1% compounded monthly. (Round answers to the nearest cent.)

\[
\begin{align*}
N &= (12 \times 30) \\
I\% &= 6.1 \\
PV &= 140000 \\
PMT &= ? - 148.39 \\
FV &= 0 \\
P/Y &= C/Y = 12
\end{align*}
\]

Find the total interest paid on the loan.

\[
N \times PMT - 140000 = (360 \times 148.39) - 140000 = $165,420.4
\]

5. Five years ago, Diane secured a bank loan of $330,000 to help finance the purchase of a loft in the San Francisco Bay area. The term of the mortgage was 30 years, and the interest rate was 10% per year compounded monthly on the unpaid balance. Because the interest rate for a conventional 30-year home mortgage has now dropped to 6% per year compounded monthly, Diane is thinking of refinancing her property. (Round answers to the nearest cent.)

\[
\begin{align*}
N &= (12 \times 30) \\
I\% &= 10 \\
PV &= 330000 \\
PMT &= ? - 2895.99 \\
FV &= 0 \\
P/Y &= C/Y = 12
\end{align*}
\]

(a) What is Diane’s current monthly mortgage payment?

\[\$ 2,895.99\]

(b) What is Diane’s current outstanding balance?

\[\text{Find } FV \text{ after 5 years.} \]
\[\text{Change } N = (12 \times 5) \]
\[FV_5 = $318,645.26\]
(c) If Diane decides to refinance her property by securing a 30-year home mortgage loan in the amount of the current outstanding principal at the prevailing interest rate of 6% per year compounded monthly, what will be her monthly mortgage payment? Use the rounded outstanding balance.

\[ \text{PMT} = \$1,910.74 \]

(d) How much less would Diane’s monthly mortgage payment be if she refinances? Use the rounded values from parts (a)-(c).

\[ \$2895.99 - 1910.74 = \$985.25 \]

6. The Turners have purchased a house for $180,000. They made an initial down payment of $20,000 and secured a mortgage with interest charged at the rate of 8%/year compounded monthly on the unpaid balance. The loan is to be amortized over 30 years. (Round answers to the nearest cent.)

\[ N = (12 \times 30) \]
\[ I\% = 8 \]
\[ PV = 160000 \]
\[ PMT = ? - 1174.02 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

(a) What monthly payment will the Turners be required to make?

\[ $1,174.02 \]

(b) What will be their equity after 10 years?

\[ \text{Equity} = \text{down payments} + (PV - FV_{10}) \text{ and off loan} \]

Find FV after 10 years

\[ FV_{10} = 140,359.53 \]

\[ \text{Equity} = 20000 + (160000 - 140,359.53) \]
\[ = \$39,640.47 \]
7. Find the amortization table for a $8,000 loan amortized in three annual payments if the interest rate is 4.7% per year compounded annually. (Round answers to the nearest cent.)

\[ N = 1 \times 3 \]
\[ I\% = 4.7 \]
\[ PV = 8000 \]
\[ PMT = 2921.17 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 1 \]

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\[ PTI_1 = (0.047/1) \times 8000 = 376 \]
\[ PTP_1 = 2921.17 - 376 = 2545.17 \]
\[ OP_1 = 8000 - 2545.17 = 5454.83 \]

\[ PTI_2 = (0.047/1) \times 5454.83 = 256.38 \]
\[ PTP_2 = 2921.17 - 256.38 = 2664.79 \]
\[ OP_2 = 5454.83 - 2664.79 = 2790.04 \]

\[ PTI_3 = (0.047/1) \times 2790.04 = 131.13 \]
\[ PTP_3 = 2921.17 - 131.13 = 2790.04 \]
\[ OP_3 = 2790.04 - 2790.04 = 0 \]