Sinking Fund Payments

With these problems we will solve for the payment $PMT$ required to obtained a given accumulated amount of money $FV$. In this case $PV$ will be zero.

1. Andrea, a self-employed individual, wishes to accumulate a retirement fund of $450,000. How much should she deposit each month into her retirement account, which pays interest at a rate of 3.5%/year compounded monthly, to reach her goal upon retirement 40 years from now? (Round answer to the nearest cent.)

\[ N = (12 \times 40) \]
\[ I\% = 3.5 \]
\[ PV = 0 \]
\[ PMT = ? - 430.76 \]
\[ FV = 450000 \]
\[ P/Y = C/Y = 12 \]

2. A corporation creates a sinking fund in order to have $540,000 to replace some machinery in 12 years. How much should be placed in this account at the end of each quarter if the annual interest rate is 4.1% compounded quarterly? (Round answers to the nearest cent.)

\[ N = (4 \times 12) \]
\[ I\% = 4.1 \]
\[ PV = 0 \]
\[ PMT = ? - 764.94 \]
\[ FV = 540000 \]
\[ P/Y = C/Y = 4 \]
Determine the value of the fund after 6 years.

\[ \text{Solve for } FV \text{ and change } N \text{ to (4X6)} \]

\[ FV = 8237,123 \]

How much interest would they earn over the life of the account?

\[ FV_{12} - N \times PMT = 54,000 - (4 \times 12)(5264.94) \]

\[ = 119,282.88 \]

**Amortization Payments** With these problems we will solve for the payment \( PMT \) required to amortize a loan of \( PV \) dollars. Amortization means we pay-off the loan so that \( FV \) is zero.

3. Carl is the beneficiary of a $28,000 trust fund set up for him by his grandparents. Under the terms of the trust, he is to receive equal installments from this fund at the end of each year over a 5-year period. If the fund earns interest at the rate of 10%/year compounded annually, what amount will he receive each year? Assume that the balance in the fund is zero after the last installment is received. (Round answer to the nearest cent.)

\[ N = (1 \times 5) \]

\[ I\% = 10 \]

\[ PV = 28,000 \]

\[ PMT = 7386.33 \]

\[ FV = 0 \]

\[ P/Y = C/Y = 1 \]

**Interest Paid** Interest paid on an amortization of a loan of \( PV \) dollars where \( N \) payments of \( PMT \) dollars have been made is given by,

\[ N \times PMT - PV \]
4. Find the monthly payment needed to amortize a typical $140,000 mortgage loan amortized over 30 years at an annual interest rate of 6.1% compounded monthly. (Round answers to the nearest cent.)

\[ N = (12 \times 30) \]
\[ I\% = 6.1 \]
\[ PV = 140,000 \]
\[ PMT = ? \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

Find the total interest paid on the loan.

\[ N \times PMT - PV = (360 \times 848.39) - 140,000 = 165,420.4 \]

5. Five years ago, Diane secured a bank loan of $330,000 to help finance the purchase of a loft in the San Francisco Bay area. The term of the mortgage was 30 years, and the interest rate was 10% per year compounded monthly on the unpaid balance. Because the interest rate for a conventional 30-year home mortgage has now dropped to 6% per year compounded monthly, Diane is thinking of refinancing her property. (Round answers to the nearest cent.)

\[ N = (12 \times 30) \]
\[ I\% = 10 \]
\[ PV = 330,000 \]
\[ PMT = ? \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

(a) What is Diane’s current monthly mortgage payment?

\[ $2,895.94 \]

(b) What is Diane’s current outstanding balance?

Find FV after 5 years.
Change \( N \) to \( (12 \times 5) \)

\[ FV = $315,645.26 \]
(c) If Diane decides to refinance her property by securing a 30-year home mortgage loan in the amount of the current outstanding principal at the prevailing interest rate of 6% per year compounded monthly, what will be her monthly mortgage payment? Use the rounded outstanding balance:

\[
\text{PMT} = 1,910.74
\]

(d) How much less would Diane’s monthly mortgage payment be if she refinances? Use the rounded values from parts (a)-(c).

\[
2895.99 - 1910.74 = 985.25
\]

6. The Turners have purchased a house for $180,000. They made an initial down payment of $20,000 and secured a mortgage with interest charged at the rate of 8%/year compounded monthly on the unpaid balance. The loan is to be amortized over 30 years. (Round answers to the nearest cent.)

\[
N = (12 \times 30) \\
I\% = 8 \\
PV = 160000 \\
PMT = ? - 1174.02 \\
FV = 0 \\
P/Y = C/Y = 12
\]

(a) What monthly payment will the Turners be required to make?

\[
\text{PMT} = 1,174.02
\]

(b) What will be their equity after 10 years?

\[
\text{Equity} = \text{down payments} + (PV - FV_{10}) \text{ paid off loan} \\
\text{Find FV after 10 years} \\
FV_{10} = 140,359.53 \\
\text{Equity} = 20000 + (160000 - 140,359.53) \\
= 39,640.47
\]
7. Find the amortization table for a $8,000 loan amortized in three annual payments if the interest rate is 4.7% per year compounded annually. (Round answers to the nearest cent.)

\[
\begin{align*}
N &= 3 \\
I\% &= 4.7 \\
PV &= 8000 \\
PMT &= -2921.17 \\
FV &= 0 \\
P/Y &= C/Y = 1
\end{align*}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{End of Period} & \text{Payment} & \text{Payment Toward Interest} & \text{Payment Toward Principal} & \text{Outstanding Principal} \\
\hline
0 & & & & 8000 \\
1 & 2921.17 & 376 & 2545.17 & 5454.83 \\
2 & 2921.17 & 256.38 & 2664.79 & 2790.04 \\
3 & 2921.17 & 131.13 & 2790.04 & 0 \\
\hline
\end{array}
\]

\[
\begin{align*}
\text{PTI}_1 &= (0.047/1) \times 8000 = 376 \\
\text{PTP}_1 &= 2921.17 - 376 = 2545.17 \\
\text{OP}_1 &= 8000 - 2545.17 = 5454.83 \\
\text{PTI}_2 &= (0.047/1) \times 5454.83 = 256.38 \\
\text{PTP}_2 &= 2921.17 - 256.38 = 2664.79 \\
\text{OP}_2 &= 5454.83 - 2664.79 = 2790.04 \\
\text{PTI}_3 &= (0.047/1) \times 2790.04 = 131.13 \\
\text{PTP}_3 &= 2921.17 - 131.13 = 2790.04 \\
\text{OP}_3 &= 2790.04 - 2790.04 = 0
\end{align*}
\]