Section 7.2 Definition of Probability

Question: Suppose we have an experiment that consists of flipping a fair 2-sided coin and observing if the coin lands on heads or tails? From section 7.1 we should know that there are 2 outcomes, and that only 1 of those outcomes is a head, but what is the probability of the coin landing on heads?

Answer:

$$\text{Probability of heads} = \frac{\# \text{ of outcomes with a head}}{\# \text{ of total outcomes}} = \frac{1}{2}$$

Probability of an Event $E$: Part I If $S$ is the sample space of an experiment with event $E$, then the probability of event $E$ occurring, written as $P(E)$, is given by

$$P(E) = \frac{n(E)}{n(S)}$$

1. A family has three children. Assuming a boy is as likely as a girl to have been born, what are the following probabilities?

(a) Three are boys and none are girls.

(b) At least 2 are girls.

$$S = \{ bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg \}$$

(a) $E = \{ bbb \}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

(b) $E = \{ ggg, ggb, gbg, bgg \}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$
2. A pair of fair 6-sided dice is rolled. What is the probability of each of the following? (Round your answers to three decimal places.)

(a) the sum of the numbers shown uppermost is less than 6

\[
P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18} = 0.278
\]

(b) at least one 3 is cast

\[
P(E) = \frac{n(E)}{n(S)} = \frac{11}{36} = 0.306
\]

3. In a sweepstakes sponsored by Gemini Paper Products, 10,000 entries have been received. If 1 grand prize is drawn, and 4 first prizes, 30 second prizes, and 550 third prizes are to be awarded, what is the probability that a person who has submitted one entry will win the following?

(a) a third prize (Round your answer to four decimal places if necessary.)

\[
P(E) = \frac{n(E)}{n(S)} = \frac{550}{10,000} = 0.055
\]

(b) any prize (Round your answer to four decimal places if necessary.)

\[
P(E) = \frac{n(E)}{n(S)} = \frac{1 + 4 + 30 + 550}{10,000} = 0.0585
\]

4. If a card is drawn at random from a standard 52-card deck, what is the probability that the card drawn is one of the following?

(a) a club

\[
P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}
\]

(b) a black card

\[
P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}
\]

(c) a king

\[
P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}
\]
5. In an attempt to study the leading causes of airline crashes, the following data were compiled from records of airline crashes (excluding sabotage and military action):

<table>
<thead>
<tr>
<th>Primary Factor</th>
<th>Accidents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight crew</td>
<td>323</td>
</tr>
<tr>
<td>Airplane</td>
<td>47</td>
</tr>
<tr>
<td>Maintenance</td>
<td>12</td>
</tr>
<tr>
<td>Weather</td>
<td>25</td>
</tr>
<tr>
<td>Airport/air traffic control</td>
<td>25</td>
</tr>
<tr>
<td>Miscellaneous/other</td>
<td>14</td>
</tr>
</tbody>
</table>

Assume that you have just learned of an airline crash and that the data give a generally good indication of the causes of airplane crashes. Give an estimate of the probability that the primary cause of the crash was due to flight crew or bad weather. (Round your answer to three decimal places.)

\[
P(E) = \frac{n(E)}{n(S)} = \frac{323 + 25}{323 + 25 + 12 + 47 + 25 + 14} = \frac{348}{446} = .780
\]

**Probability Distribution** If \(S = \{s_1, s_2, \cdots, s_n\}\) is the sample space of a given experiment, then the probability distribution is a table where the entries in the first row are all the outcomes of \(S\) and the entries in the second row are all their corresponding probabilities.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(\cdots)</th>
<th>(s_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>(P(s_1))</td>
<td>(P(s_2))</td>
<td>(\cdots)</td>
<td>(P(s_n))</td>
</tr>
</tbody>
</table>

6. The grade distribution for a certain class is shown in the following table. Find the probability distribution associated with these data. (Enter your answers to three decimal places.)

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of Occurrence</td>
<td>5</td>
<td>10</td>
<td>16</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Probability</td>
<td>.125</td>
<td>.25</td>
<td>.4</td>
<td>.125</td>
<td>.05</td>
</tr>
</tbody>
</table>

\[
P(A) = \frac{n(A)}{n(S)} = \frac{5}{40} = .125, \quad P(B) = \frac{10}{40} = .25
\]

\[
P(C) = \frac{16}{40} = .4, \quad P(D) = \frac{7}{40} = .175, \quad P(E) = \frac{2}{40} = .05
\]
7. In a survey conducted by a business-advisory firm of 4980 adults 18 years old and older in June 2009, during the “Great Recession,” the following question was asked: How long do you think it will take to recover your personal net worth? The results of the survey follow. (Round your answers to three decimal places.)

<table>
<thead>
<tr>
<th>Answer (years)</th>
<th>1 – 2</th>
<th>3 – 4</th>
<th>5 – 10</th>
<th>≥ 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Respondents</td>
<td>1004</td>
<td>1301</td>
<td>2114</td>
<td>561</td>
</tr>
</tbody>
</table>

(a) Determine the empirical probability distribution associated with these data.

<table>
<thead>
<tr>
<th>Answer (years)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2</td>
<td>.202</td>
</tr>
<tr>
<td>3 – 4</td>
<td>.261</td>
</tr>
<tr>
<td>5 – 10</td>
<td>.425</td>
</tr>
<tr>
<td>≥ 10</td>
<td>.113</td>
</tr>
</tbody>
</table>

\[ P(1-2) = \frac{1004}{4980} = .202, \quad P(3-4) = \frac{1301}{4980} = .261 \]
\[ P(5-10) = \frac{2114}{4980} = .425, \quad P(≥10) = \frac{561}{4980} = .113 \]

(b) If a person who participated in the survey is selected at random, what is the probability that he or she expects that it will take 5 or more years to recover his or her personal net worth?

\[ P(5 \text{ or more years}) = .425 + .113 = .538 \]

Properties of Probability Distributions

1. \( 0 \leq P(s_i) \leq 1 \) for \( i = 1, 2, \cdots, n \).
2. \( P(s_1) + P(s_2) + \cdots + P(s_n) = 1 \)
3. \( P(\{s_i\} \cup \{s_i\}) = P(s_i) + P(s_i) \) (\( i \neq j \)) for \( i = 1, 2, \cdots, n; j = 1, 2, \cdots, n \).

Finding the Probability of an Event \( E \): Part II If \( E = \{s_1, s_2, \cdots, s_m\} \) is an event of a sample space \( S \) of a given experiment, then

\[ P(E) = P(s_1) + P(s_2) + \cdots + P(s_m) \]

If \( E \) is the empty set, \( \emptyset \), then \( P(E) = 0 \).
8. A die is loaded, and it has been determined that the probability distribution associated with the experiment of rolling the die and observing which number falls uppermost is given by the following:

<table>
<thead>
<tr>
<th>Simple Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>.18</td>
</tr>
<tr>
<td>{2}</td>
<td>.13</td>
</tr>
<tr>
<td>{3}</td>
<td>.19</td>
</tr>
<tr>
<td>{4}</td>
<td>.2</td>
</tr>
<tr>
<td>{5}</td>
<td>.15</td>
</tr>
<tr>
<td>{6}</td>
<td>.15</td>
</tr>
</tbody>
</table>

(a) What is the probability of the number being even?

\[ P(\text{Even}) = P(1) + P(2) + P(4) + P(6) = .18 + .13 + .19 + .15 = .65 \]

(b) What is the probability of the number being either a 1 or a 6?

\[ P(1 \cup 6) = .18 + .15 = .33 \]

(c) What is the probability of the number being less than 4?

\[ P(<4) = P(1) + P(2) + P(3) = .18 + .13 + .19 = .50 \]
9. Let \( S = \{s_1, s_2, s_3, s_4, s_5, s_6\} \) be the sample space associated with the experiment having the following probability distribution. (Enter your answers as fractions.)

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
<th>( s_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{3}{12} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{4}{12} )</td>
<td>( \frac{2}{12} )</td>
</tr>
</tbody>
</table>

(a) Find the probability of \( A = \{s_1, s_3\} \).

\[
P(A) = P(s_1) + P(s_3) = \frac{1}{12} + \frac{3}{12} = \frac{4}{12} = \frac{1}{3}
\]

(b) Find the probability of \( B = \{s_2, s_4, s_5, s_6\} \).

\[
P(B) = P(s_2) + P(s_4) + P(s_5) + P(s_6) = \frac{1}{12} + \frac{1}{12} + \frac{4}{12} + \frac{2}{12} = \frac{8}{12} = \frac{2}{3}
\]

(c) Find the probability of \( C = S \).

\[
P(S) = P(s_1) + P(s_2) + P(s_3) + P(s_4) + P(s_5) + P(s_6) = 1
\]

10. The sample space \( S = \{s_1, s_2, s_3\} \) has the property that \( P(s_1) + P(s_2) = 0.39 \) and \( P(s_2) + P(s_3) = 0.83 \). Find the probability of each outcome.

\[
P(s_1) + P(s_2) + P(s_3) = 1
\]

\[
.39 + P(s_3) = 1 \quad \Rightarrow \quad P(s_3) = .61
\]

\[
P(s_2) + P(s_3) = .83
\]

\[
P(s_2) + .61 = .83 \quad \Rightarrow \quad P(s_2) = .22
\]

\[
P(s_1) + P(s_2) = .59
\]

\[
P(s_1) = .39 - .22 = .17
\]