Section 5.3 Amortization

Present Value of an Annuity The present value $P$ of an annuity consisting of $n$ payments of $R$ dollars each, paid at the end of each investment period into an account that earns interest at the rate of $i$ per period, is

$$P = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$

Note: The future value $S$ does not appear in the above formula. This means that when using the TVM solver that the entry “FV” will be zero.

1. Find the amount needed to deposit into an account today that will yield pension payments of $35,000 at the end of each of the next 29 years if the account earns interest at a rate of 5.9%/yr compounded annually. (Round answer to the nearest cent.)

$$N = 1(29)$$
$$I\% = 5.9$$
$$PV = ? 480,699.44$$
$$PMT = -35,000$$
$$FV = 0$$
$$P/Y = C/Y = 1$$

2. A local moving service recently purchased a van by securing a loan with semiannual payments of $2900 per semiannual period for 6 years at 12% per year compounded semiannually. What was the purchase price of this van? (Round answer to the nearest cent.)

$$N = 2(6)$$
$$I\% = 12$$
$$PV = ? 24,313.15$$
$$PMT = -2960$$
$$FV = 0$$
$$P/Y = C/Y = 2$$
3. Lupe made a down payment of $2200 toward the purchase of a new car. To pay the balance of the purchase price, she has secured a loan from her bank at the rate of 13%/year compounded monthly. Under the terms of her finance agreement she is required to make payments of $240/month for 48 months. What is the cash price of the car? (Round your answer to the nearest cent.)

\[
\begin{align*}
N &= 12(4) \\
I% &= 13 \\
PV &= 8946.05 \\
PMT &= -240 \\
FV &= 0 \\
P/Y = C/Y &= 12
\end{align*}
\]

Amortization Payments With these problems we will solve for the payment \( PMT \) required to amortize a loan of \( PV \) dollars. Amortization means we pay-off the loan so that \( FV \) is zero.

4. Carl is the beneficiary of a $28,000 trust fund set up for him by his grandparents. Under the terms of the trust, he is to receive equal installments from this fund at the end of each year over a 5-year period. If the fund earns interest at the rate of 10%/year compounded annually, what amount will he receive each year? Assume that the balance in the fund is zero after the last installment is received. (Round answer to the nearest cent.)

\[
\begin{align*}
N &= 1(5) \\
I% &= 10 \\
PV &= 28000 \\
PMT &= 7386.53 \\
FV &= 0 \\
P/Y = C/Y &= 1
\end{align*}
\]

Interest Paid Interest paid on an amortization of a loan of \( PV \) dollars where \( N \) payments of \( PMT \) dollars have been made is given by,
5. Find the monthly payment needed to amortize a typical $140,000 mortgage loan amortized over 30 years at an annual interest rate of 6.1\% compounded monthly. (Round answers to the nearest cent.)

\[ N = 12(30) \]
\[ I\% = 6.1 \]
\[ PV = 140,000 \]
\[ PMT = ? \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

Find the total interest paid on the loan.

\[ N \times PMT - PV = \]
\[ (260 \times 848.39) - 140,000 = $165,420.40 \]

6. Five years ago, Diane secured a bank loan of $330,000 to help finance the purchase of a loft in the San Francisco Bay area. The term of the mortgage was 30 years, and the interest rate was 10\% per year compounded monthly on the unpaid balance. Because the interest rate for a conventional 30-year home mortgage has now dropped to 6\% per year compounded monthly, Diane is thinking of refinancing her property. (Round answers to the nearest cent.)

\[ N = 12(30) \]
\[ I\% = 10 \]
\[ PV = 330,000 \]
\[ PMT = ? \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

(a) What is Diane’s current monthly mortgage payment?

\[ PMT = $2,895.99 \]

(b) What is Diane’s current outstanding balance?

\[ FV_{5\;\text{yrs}} = \]
\[ $318,695.24 \]
\[ N = 12(5) \]
(c) If Diane decides to refinance her property by securing a 30-year home mortgage loan in the amount of the current outstanding principal at the prevailing interest rate of 6% per year compounded monthly, what will be her monthly mortgage payment? Use the rounded outstanding balance.

\[ PMT_{6\%} = \$1,910.74 \]

(d) How much less would Diane’s monthly mortgage payment be if she refinances? Use the rounded values from parts (a)-(c).

\[ PMT_{10\%} - PMT_{6\%} = \$2,189.95 - \$1,910.74 = \$989.25 \]

(e) How much less interest would she have paid if she started out with the lower monthly payment?

\[
\left( N_{30\text{ yrs}} \cdot PMT_{10\%} - 330000 \right) - \left( N_{30\text{ yrs}} \cdot PMT_{6\%} - 318695.26 \right) = \$343,285.26
\]

**Equity:** A person’s equity in a home is how much of the home’s value they have already invested in the home.

\[
\frac{(\text{Loan Amount Paid Off}) + (\text{Down payments})}{\text{OR}} \quad \frac{\text{(Total Home Value)} - (\text{Outstanding Balance})}{\text{OR}}
\]

7. Suppose, in the last example, that Diane made a down payment of $15,000 towards the purchase of the loft, making the loft’s value $345,000. How much equity does she have when she goes to refinance?

\[
345000 - FV_{5\text{ yrs}} = 345000 - 318695.26 = \$26,304.74
\]

\[
(330000 - 318695.26) + 15000 = \$26,304.74
\]
8. The Turners have purchased a house for $180,000. They made an initial down payment of $20,000 and secured a mortgage with interest charged at the rate of $8\%/\text{year}$ compounded monthly on the unpaid balance. The loan is to be amortized over 30 years. (Round answers to the nearest cent.)

\[ N = 12 \times 30 \]
\[ I\% = 8 \]
\[ PV = 180000 - 20000 \]
\[ PMT = \, ? - 1174.02 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

(a) What monthly payment will the Turners be required to make?

\[ PMT = 1174.02 \]

(b) What will be their equity after 10 years?

\[ \text{Home value} - \text{Outstanding Balance} = FV_{10 \text{ yrs}} \]
\[ 180000 - 140359.53 = 39,640.47 \]
9. Find the amortization table for a $8,000 loan amortized in three annual payments if the interest rate is 4.7% per year compounded annually. (Round answers to the nearest cent.)

\[ N = 1(3) \]
\[ I\% = 4.7 \]
\[ PV = 8000 \]
\[ PMT = ? - 2921.17 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 1 \]

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\[ PTI_1 = (0.047/1) \times 8000 = 376 \]
\[ PTI_2 = 2921.17 - 376 = 2545.17 \]
\[ OP_1 = 8000 - 2545.17 = 5454.83 \]

\[ PTI_2 = (0.047/1) \times 5454.83 = 256.38 \]
\[ PTI_3 = 2921.17 - 256.38 = 2664.79 \]
\[ OP_2 = 5454.83 - 2664.79 = 2790.04 \]

\[ PTI_3 = (0.047/1) \times 2790.04 = 131.13 \]
\[ PTI_4 = 2921.17 - 131.13 = 2790.04 \]
\[ OP_3 = 2790.04 - 2790.04 = 0 \]