1. A die is loaded, and it has been determined that the probability distribution associated with the experiment of rolling the die and observing which number falls uppermost is given by the following:

<table>
<thead>
<tr>
<th>Simple Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>.18</td>
</tr>
<tr>
<td>{2}</td>
<td>.13</td>
</tr>
<tr>
<td>{3}</td>
<td>.19</td>
</tr>
<tr>
<td>{4}</td>
<td>.2</td>
</tr>
<tr>
<td>{5}</td>
<td>.15</td>
</tr>
<tr>
<td>{6}</td>
<td>.15</td>
</tr>
</tbody>
</table>

(a) What is the probability of the number being even?

\[
.13 + .15 + .15 = \frac{48}{120}
\]

(b) What is the probability of the number being either a 1 or a 6?

\[
.18 + .15 = \frac{33}{120}
\]

(c) What is the probability of the number being less than 4?

\[
.18 + .13 + .19 = \frac{5}{5}
\]

2. Let \( S = \{s_1, s_2, s_3, s_4, s_5, s_6\} \) be the sample space associated with the experiment having the following probability distribution. (Enter your answers as fractions.)

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
<th>( s_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{3}{12} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{4}{12} )</td>
<td>( \frac{2}{12} )</td>
</tr>
</tbody>
</table>

(a) Find the probability of \( A = \{s_1, s_3\} \).

\[
\frac{1}{12} + \frac{3}{12} = \frac{4}{12}
\]

(b) Find the probability of \( B = \{s_2, s_4, s_5, s_6\} \).

\[
\frac{1}{12} + \frac{1}{12} + \frac{4}{12} + \frac{2}{12} = \frac{8}{12}
\]

(c) Find the probability of \( C = S \).

\[
P(S) = 1
\]
3. An experiment consists of selecting a card at random from a 52-card deck. Refer to this experiment and find the probability of the following events

(a) A spade or a Jack is drawn
\[
\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}
\]

(b) A black card or a 2 is drawn
\[
\frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{17}{13}
\]

(c) A heart or a face card is drawn
\[
\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}
\]

4. Let \( S = \{s_1, s_2, s_3, s_4, s_5, s_6\} \) be the sample space associated with an experiment having the following partial probability distribution.

\[
\begin{array}{cccccc}
\text{s} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\
P(s) & \frac{4}{29} & \frac{10}{29} & \frac{2}{29} & \frac{3}{29} & \frac{2}{29} \\
\end{array}
\]

Consider the events: \( A = \{s_1, s_2, s_5\} \), \( B = \{s_3, s_5, s_6\} \), \( C = \{s_1, s_3, s_4, s_6\} \), and \( D = \{s_1, s_2, s_3\} \)

Calculate the following probabilities. (Give answers as fractions.)

(a) \( P(s_2) \)
\[
\frac{\frac{29}{29} - 2 \frac{1}{29}}{\frac{29}{29}} = \frac{8}{29}
\]

(b) \( P(D) \)
\[
\frac{\frac{4}{29} + \frac{8}{29} + \frac{10}{29}}{29} = \frac{22}{29}
\]

(c) \( P(B^c) \)
\[
\frac{\frac{4}{29} + \frac{8}{29} + \frac{2}{29}}{29} = \frac{14}{29}
\]

(d) \( P(A^c \cap B) \)
\[
\frac{\frac{10}{29} + \frac{2}{29}}{29} = \frac{12}{29}
\]

(e) \( P(C^c \cup D) \)
\[
\frac{\frac{4}{29} + \frac{8}{29} + \frac{10}{29} + \frac{3}{29}}{29} = \frac{25}{29}
\]
5. Let $E$ and $F$ be two events of an experiment with sample space $S$. Suppose $P(E) = 0.59$, $P(F) = 0.38$, and $P(E^c \cap F) = 0.28$. Calculate the probabilities below.

(a) $P(E^c) = 1 - P(E) = 1 - 0.59 = 0.41$

(b) $P(E \cap F) = 0.10$

(c) $P(E \cup F) = 0.59 + 0.38 - 0.10 = 0.87$

(d) $P(E^c \cap F^c) = 0.13$

(e) $P(E \cap F^c) = 0.59 + 0.13 = 0.72$

6. Five marbles are selected at random without replacement from a jar containing four white marbles and six blue marbles. Find the probability of the given events. (Round answer to three decimal places.)

(a) Three of the marbles are white.

$$\frac{\binom{4}{3} \cdot \binom{6}{2}}{\binom{10}{5}} = 0.238$$

(b) Two of the marbles are blue.

$$\frac{\binom{6}{2} \cdot \binom{4}{3}}{\binom{10}{5}} = 0.238$$

(c) None of the marbles are white.

$$\frac{\binom{6}{5}}{\binom{10}{5}} = 0.024$$
7. From the tree diagram find the following.

(a) \( P(A \cap E) = \)
\[ (0.7)(0.1) = 0.07 \]

(b) \( P(A) = \)
\[ (0.7)(0.1) + (0.3)(0.1) = 0.1 \]

(c) \( P(A|E) = \frac{P(A \cap E)}{P(E)} = (0.7)(0.1) \]
\[ \frac{0.07}{0.1} = 0.1 \]

8. Two machines turn out all the products in a factory, with the first machine producing 75% of the product and the second 25%. The first machine produces defective products 5% of the time and the second machine 7% of the time.

(a) Construct a tree diagram from the given information

(b) What is the probability that a defective part is produced at this factory given that it was made on the first machine?
\[ P(D|M1) = \frac{P(D \cap M1)}{P(M1)} = \frac{(0.75)(0.05)}{0.75} = 0.05 \]
(c) What is the probability that a defective part is produced at this factory?

\[ P(D) = (0.75)(0.05) + (0.25)(0.07) \]

\[ = 0.055 \]

9. Suppose \( A \) and \( B \) are two events of a sample space \( S \) where \( P(A) = 0.28 \), \( P(B) = 0.24 \), and \( P(A \cup B) = 0.42 \).

(a) What is \( P(A \cap B) \)?

\[ 0.42 = 0.28 + 0.24 - P(A \cap B) \]

\[ P(A \cap B) = 0.1 \]

(b) Are \( A \) and \( B \) independent events?

\[ P(A)P(B) = (0.28)(0.24) = 0.0672 \]

\[ P(A \cap B) = 0.1 \]

\[ P(A \cap B) \neq P(A)P(B) \]

So not independent!

10. If \( A \) and \( B \) are independent events, \( P(A) = 0.35 \), and \( P(B) = 0.55 \), find the probabilities below. (Enter answers to four decimal places.)

(a) \( P(A \cap B) = (0.35)(0.55) = 0.1925 \)

(b) \( P(A \cup B) = 0.35 + 0.55 - 0.1925 = 0.7075 \)

(c) \( P(A^c \cap B^c) = (1 - 0.35)(1 - 0.55) = 0.2925 \)

(d) \( P(A^c \cup B^c) \)

\[ = (1 - 0.35) + (1 - 0.55) - 0.2925 \]

\[ = 0.8075 \]
11. Find \( P(F|B) \) and \( P(E|A) \) using the tree diagram. (Round answers to three decimal places.)

\[
P(F|B) = \frac{P(F \cap B)}{P(B)} = \frac{0.476}{0.67} = 0.711
\]

\[
P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{0.35}{0.35 + 0.25 + 0.15} = 0.351
\]

12. Three machines turn out all the products in a factory, with the first machine producing 35% of the products, the second machine 25%, and the third machine 40%. The first machine produces defective products 6% of the time, the second machine 17% of the time and the third machine 4% of the time.

(a) Construct a tree diagram from the given information

(b) What is the probability that a non-defective product came from the second machine? (Round answer to four decimal places.)

\[
P(M_2|ND) = \frac{P(M_2 \cap ND)}{P(ND)} = \frac{0.25 \times 0.83}{0.25 \times 0.83 + 0.35 \times 0.94 + 0.40 \times 0.46} = 0.2254
\]

13. A medical test has been designed to detect the presence of a certain disease. Among people who have the disease, the probability that the disease will be detected by the test is 0.91. However, among those who do not have the disease, the probability that the test will detect the presence of the disease is 0.04. It is estimated that 3% of the population who take this test actually have the disease. (Round answers to three decimal places.)

(a) Construct a tree diagram from the given information
(b) If the test administered to an individual is positive (the disease is detected), what is the probability that the person actually has the disease?

\[
P(H|D) = \frac{P(H \cap D)}{P(D)} = \frac{(.03 \cdot .91)}{(.03 \cdot .91) + (.97 \cdot .04)}
\]

\[= .413\]

14. For each of the following experiments, give the range for the random variable \(X\) and classify it as finite discrete, infinite discrete or continuous.

(a) \(X = \) The number of times a die is thrown until a 2 appears

\[X = \{1, 2, 3, \ldots\}; \ X \text{ is infinite discrete.}\]

(b) \(X = \) The number of hours a child watches television on a given day

\[X = \{x \mid 0 \leq x \leq 24\}; \ X \text{ is continuous}\]

(c) Cards are selected one at a time without replacement from a well-shuffled deck of 52 cards until a queen is drawn. Let \(X\) denote the random variable that gives the number of cards drawn.

\[X = \{1, 2, 3, \ldots, 49\}; \ X \text{ is finite discrete.}\]

15. Determine which of the follow tables represent valid probability distributions of a random variable \(X\).

(a) \[
\begin{array}{c|cccc}
\hline
x & 0 & 1 & 2 & 3 & 4 \\
\hline
P(X = x) & .6 & .1 & .2 & .1 & .3 \\
\hline
\end{array}
\]

\[.6 + .1 + .2 + .1 + .3 = 1\]

Not valid!

(b) \[
\begin{array}{c|cccc}
\hline
x & -2 & -1 & 0 & 1 & 2 \\
\hline
P(X = x) & .4 & .1 & .2 & .1 & .2 \\
\hline
\end{array}
\]

\[.4 + .1 + .2 + .1 + .2 = 1\]

Valid!

(c) \[
\begin{array}{c|ccccc}
\hline
x & -150 & 250 & 350 & 450 & 500 \\
\hline
P(X = x) & .01 & .04 & .04 & .3 & .3 \\
\hline
\end{array}
\]

\[.01 + .04 + .04 + .3 + .3 = 1\]

Not valid!
16. Two cards are drawn from a well-shuffled deck of 52 playing cards. Let $X$ denote the number of aces drawn. Construct the probability distribution of the random variable $X$. (Round answer to three decimal places.)

$$
P(X = 0) = \frac{C(48, 2)}{C(52, 2)} = .851 \quad P(X = 1) = \frac{C(4, 1) \cdot C(48, 1)}{C(52, 2)} = .145 \quad P(X = 2) = \frac{C(4, 2)}{C(52, 2)} = .005
$$

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>.851</td>
<td>.145</td>
<td>.005</td>
</tr>
</tbody>
</table>

17. A box has 5 yellow, 7 gray, and 3 black marbles. Three marbles are drawn one at a time without replacement from the box. Let $X$ be the number of gray marbles drawn. Construct the probability distribution of the random variable $X$. (Round answer to three decimal places.)

$$
P(X = 0) = \frac{C(8, 3)}{C(15, 3)} = .123 \quad P(X = 1) = \frac{C(7, 1) \cdot C(8, 2)}{C(15, 3)} = .431 \quad P(X = 2) = \frac{C(7, 2) \cdot C(8, 1)}{C(15, 3)} = .369
$$

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>.123</td>
<td>.431</td>
<td>.369</td>
<td>.077</td>
</tr>
</tbody>
</table>

18. A man purchased a $20,000, 1-year term-life insurance policy for $390. Assuming that the probability that he will live for another year is 0.988, find the company’s expected net gain.

$$
\begin{array}{c|c|c|c|c|}
\hline
x & 390 & 390 - 20000 \\
\hline
p(x) & .988 & .012 \\
\hline
\end{array}
$$

$$
E(X) = .988(390) + .012(390 - 20000) = \$150
$$

19. A man wishes to purchase a life insurance policy that will pay the beneficiary $25,000 in the event that the man’s death occurs during the next year. Using life insurance tables, he determines that the probability that he will live another year is 0.96. What is the minimum amount that he can expect to pay for his premium? **Hint:** The minimum premium occurs when the insurance company’s expected profit is zero.

$$
\begin{array}{c|c|c|}
\hline
x & y & y - 25000 \\
\hline
p(x) & .96 & .04 \\
\hline
\end{array}
$$

$$
E(X) = .96y + .04(y - 25000) = 0
$$

$$
\Rightarrow y - 1000 = 0
$$

$$
\Rightarrow y = 1000
$$

Minimum payment is $1000
20. Find the following:

(a) Given \( P(A) = \frac{2}{9} \), find odds in favor and odds against \( A \) occurring.

Odds in favor: \( 2 \) to \( 7 \)
Odds against: \( 7 \) to \( 2 \)

(b) Given \( P(A) = \frac{21}{50} \), find odds in favor and odds against \( A \) occurring.

Odds in favor: \( 21 \) to \( 29 \)
Odds against: \( 29 \) to \( 21 \)

(c) Given \( P(A) = \frac{3}{5} \), find odds in favor and odds against \( A \) occurring.

Odds in favor: \( 3 \) to \( 2 \)
Odds against: \( 2 \) to \( 3 \)

21. Find the following:

(a) If the odds in favor of you winning a game are 11 to 31, find the probability that you will win the game and the probability that you will lose the game.

\[
P(Win) = \frac{11}{42}; \quad P(Lose) = \frac{31}{42}\]

(b) If the odds against you winning a game (odds that you lose) are 6 to 13, find the probability that you will win the game and the probability that you will lose the game.

\[
P(Win) = \frac{13}{19}; \quad P(Lose) = \frac{6}{19}\]

22. Find the mean, median, mode, standard deviation, and variance of the probability distribution for the histogram shown. (Round answer to two decimal places.)

Mean = 4.1, median = 4,
\( \sigma^2 = 2.39 \), mode = 1,
\( \text{Var}(X) = 5.71 \)
23. The distribution of the number of chocolate chips \( (x) \) in a cookie is shown in the following table.

\[
\begin{array}{c|cccccccc}
   & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
L1 & x & 0.01 & 0.04 & 0.05 & 0.11 & 0.12 & 0.24 & 0.22 & 0.17 & 0.04 \\
L2 & P(X=x) & & & & & & & & & \\
\end{array}
\]

Find the mean, median, mode, standard deviation, and variance of the number of chocolate chips in a cookie. (Round answers to two decimal places.)

\[
\text{Mean } = 4.98, \quad \text{Median } = 5, \quad \text{Mode } = 5,
\]

\[
\sigma_x = 1.78, \quad \text{Var}(X) = 3.17
\]

24. A panel of 76 economists was asked to predict the average unemployment rate for the upcoming year. The results of the survey follow.

\[
\begin{array}{c|cccccccc}
\hline
   & 10.7 & 11.1 & 11.5 & 11.9 & 12.3 & 12.7 & 13.1 & \\
\hline
L1 & \text{Unemployment Rate, } \% & 3 & 6 & 8 & 24 & 18 & 12 & 5 \\
L2 & \text{Economists} & & & & & & & \\
\end{array}
\]

Find the mean, median, mode, standard deviation, and variance of the average unemployment rate for the upcoming year. (Round answer to two decimal places.)

\[
\text{Mean } = 12.05, \quad \text{Median } = 11.9, \quad \text{Mode } = 11.9,
\]

\[
\sigma_x = .58, \quad \text{Var}(X) = .34
\]

25. It is estimated that one third of the general population has blood type A+. A sample of six people is selected at random. (Round answers to four decimal places.)

\[
n = 6, \quad p = \frac{1}{3}
\]

(a) What is the probability that exactly two of them have blood type A+?

\[
P(X=2) = \text{Binomial pdf}(6, \frac{1}{3}, 2) = .3292
\]

(b) What is the probability that at most two of them have blood type A+?

\[
P(X \leq 2) = \text{Binomial cdf}(6, \frac{1}{3}, 2) = .6804
\]
26. The manager of Toy World knows that the probability an electronic game will be returned to the store is 0.22. If 54 games are sold in a given week, determine the probabilities of the following events. (Round answers to four decimal places.)

(a) No more than 12 games will be returned.
\[ P(X \leq 12) = \text{Binomcdf}(54, 0.22, 12) = 0.5921 \]
(b) At least 8 games will be returned.
\[ P(X \geq 8) = 1 - \text{Binomcdf}(54, 0.22, 7) = 0.9307 \]
(c) More than 5 games but fewer than 14 games will be returned.
\[ P(5 < X < 14) = \text{Binomcdf}(54, 0.22, 13) - \text{Binomcdf}(54, 0.22, 5) = 0.6971 \]

27. A coin is biased so that the probability of tossing a head is 0.46. If this coin is tossed 54 times, determine the probabilities of the following events. (Round answers to four decimal places.)

(a) The coin lands heads more than 21 times.
\[ P(X > 21) = 1 - \text{Binomcdf}(54, 0.46, 21) = 0.8189 \]
(b) The coin lands heads fewer than 28 times.
\[ P(X < 28) = \text{Binomcdf}(54, 0.46, 27) = 0.7665 \]
(c) The coin lands heads at least 20 times but at most 27 times.
\[ P(20 \leq X \leq 27) = \text{Binomcdf}(54, 0.46, 27) - \text{Binomcdf}(54, 0.46, 19) = 0.6951 \]

28. A company finds that one out of three employees will be late to work on a given day. If this company has 60 employees, how many employees can they expect to be late to work on a given day?
\[ E(x) = \rho n = \left(\frac{1}{3}\right)(60) = 20 \text{ people} \]

29. A new drug has been found to be effective in treating 70% of the people afflicted by a certain disease. If the drug is administered to 500 people who have this disease, what are the mean, variance, and the standard deviation of the number of people for whom the drug can be expected to be effective? (Round answers to two decimal places.)
\[ \text{Mean } = \rho n = (0.7)(500) = 350, \]
\[ \text{Var}(X) = n\rho(1-\rho) = (0.7)(0.3)(500) = 105, \text{ and } \sigma_X = \sqrt{105} = 10.25 \]

30. A 6-sided die is loaded so that the probability of rolling a 3 is 2/7. If you roll this die 600 times, how many times can you expect the die to land on 3?
\[ E(X) = \rho n = \left(\frac{2}{7}\right)(600) = 171.43 \]
\[ \text{rounded to nearest integer } \boxed{171} \text{ times} \]