MATH 142–511, 516, 517 Spring 2010

Sample problems for Test 2

1. Let
$$p = 25 - 0.01x \quad \text{and} \quad C(x) = 2x + 9000$$
where $0 \le x \le 2500$, be the price-demand equation and cost function, respectively, for the manufacture of umbrellas.

   (a) Find the exact cost of producing the 31st umbrella. Use the marginal cost to approximate the cost of producing the 31st umbrella.

   (b) Find the marginal revenue and the marginal average revenue functions.

   (c) Find the average profit per umbrella if 20 umbrellas is produced. Find the marginal average profit at a production level of 20 umbrellas. Estimate the average profit per umbrella if 21 umbrella is produced.

2. A bank offers a 10-year certificate of deposit (CD) that earns 4.15% compounded continuously.

   (a) If \$10000 is invested in this CD, how much will it be worth in 10 years?

   (b) How long will it take for the account to be worth \$18000?

3. A note will pay \$25000 at maturity 10 years from now. How much should you willing to pay for the note now if money is worth 5% compounded continuously?

4. At what nominal rate compounded continuously must money be invested to double in 8 years?

5. How long will it take for the U.S. population to double if it is continues to grow at a rate of 0.975% per year?

6. Find the equation of the tangent line to the graph of the function $f(x) = \ln(1 - x^2 + 2x^4)$ at the point where $x = 1$.

7. Find the value(s) of x where the tangent line to the graph of the function $y = 5e^{x^2-4x+1}$ is horizontal.

8. Find each derivative

   (a) $\dfrac{d}{dx}\log_3(\sqrt[4]{4x^3 + 5x + 7})$

   (b) $\dfrac{d}{dx}8^{1-2x^3}$

   (c) $\dfrac{d}{dx}\dfrac{3x^2}{(x^2+5)^3}$

   (d) $\dfrac{d}{dx}[(x^2 + x - 3)e^{2x+3}]$

9. Given the price-demand equation

$$0.02x + p = 60$$

   (a) Find the elasticity of demand $E(p)$.

   (b) For which values of p is demand elastic?

   (c) If $p = \$10$ and the price is increased by 5%, what is the approximate change in demand?

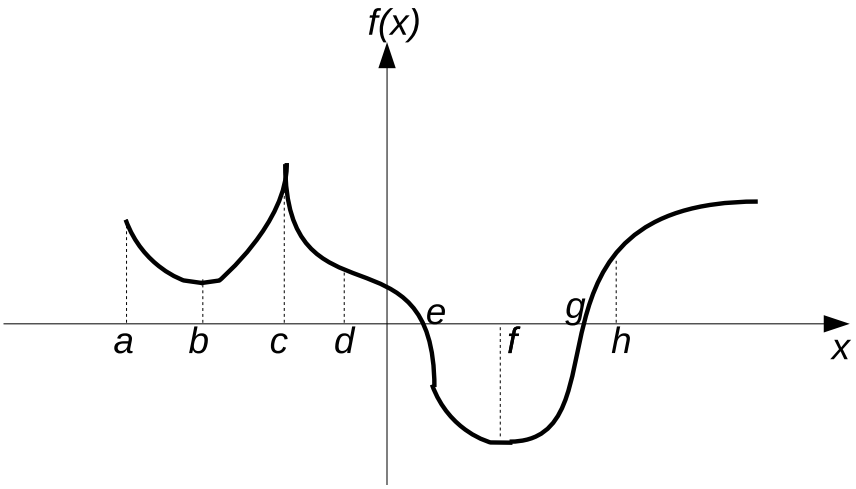
   (d) If $p = \$40$ and the price is decreased, will revenue increase or decrease?

10. Find $f''(x)$ for the functions

   (a) $f(x) = x^2(2x^3 - 5)^4$

   (b) $f(x) = \dfrac{2}{x} - \dfrac{6}{x^3}$

11. Given the graph of the function $y = f(x)$.



   (a) Find the intervals on which $f'(x) > 0$.

   (b) Find the intervals on which $f'(x) < 0$.

   (c) Find x-coordinates of the points where $f'(x) = 0$.

   (d) Find the intervals on which $f''(x) > 0$.

   (e) Find the intervals on which $f''(x) < 0$.

   (f) Find x-coordinates of the points where $f''(x) = 0$.

12. Given the function $f(x) = \dfrac{1}{4}x^4 - 4x$.

   (a) Find critical values of $f(x)$.

   (b) Find intervals on which $f(x)$ is increasing and decreasing.

(c) Find local extrema for $f(x)$.

(d) Find intervals on which $f(x)$ is concave upward and concave downward.

(e) Find all inflection points of $f(x)$.

13. Find the absolute maximum and absolute minimum for the function $f(x) = \sqrt{9 - x^2}$ on the interval $[-1, 2]$.

14. Find the absolute maximum and minimum for the function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

15. A box with a square base and open top must have a volume of 32000 cm^3. Find the dimensions of the box that minimize the amount of material used.