

**Sample problems for Test 3**

1. Find the following indefinite integrals:

(a)  $\int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$

(b)  $\int \frac{x^3 - 3}{x^4 - 12x + 3} dx$

(c)  $\int \frac{(\ln x)^2}{x} dx$

(d)  $\int \frac{x}{(5 - 2x^2)^5} dx$

(e)  $\int \frac{x}{\sqrt{3-x}} dx$

(f)  $\int e^{2x-1} dx$

2. The weekly marginal revenue from the sale of  $x$  pairs of tennis shoes is given by

$$R'(x) = 40 - 0.002x + \frac{200}{x+1}, \quad R(0) = 0,$$

where  $R(x)$  is revenue in dollars. Find the revenue function  $R(x)$ . Find the revenue from the sale of 1000 pairs of shoes.

3. Use  $L_6$  and  $R_6$  to approximate  $\int_2^5 (0.25x^2 - 4) dx$ .

4. Evaluate each of the definite integrals:

(a)  $\int_0^A 32(x^2 + 1)^7 x dx \quad (A > 0)$

(b)  $\int_B^2 \left( 5x - 4 \frac{x^2}{\sqrt[4]{x^3}} \right) dx, \quad (0 < B < 2)$

5. Find the average value of the function  $f(x) = 4x - 3x^2$  over the interval  $[-2, 2]$ .

6. Find the area of the region bounded by:

(a)  $y = 3 - x^2, y = 2x^2 - 4x$

(b)  $y = x^3, y = 4x$

(c)  $y = -x^2 - 2x, y = 0, x = -2, x = 1$ .

7. Find the consumers' surplus and producers' surplus at the equilibrium price level for the given price-demand and price-supply equations.

$$\begin{aligned}p &= D(x) = 70 - 0.2x \\p &= S(x) = 13 + 0.0012x^2\end{aligned}$$

Round all values to the nearest integer.

8. Let  $f(x, y) = 2x - 3y + 14$  and  $g(x, y) = \frac{10}{x^2 + 4y}$ . Find  $f(2, -3) - 4g(-1, 2)$ .
9. Weston Publishing publishes a deluxe edition and a standard edition of its English language dictionary. Weston's management estimates that the number of deluxe editions demanded is  $x$  copies/day and the number of standard editions demanded is  $y$  copies/day when the unit prices are

$$\begin{aligned}p &= 20 - 0.005x - 0.001y \\q &= 15 - 0.001x - 0.003y\end{aligned}$$

dollars, respectively. Find the daily total revenue function  $R(x, y)$ . Evaluate  $R(7, 3)$ .

10. Find the cross-section of the surface  $z = 10x + 4xy + 15y^2 - 6x^2 + 5$  produced by the cutting it with the planes  $x = 4$ ,  $y = 2$ .
11. Find  $f_x$  and  $f_y$  for the functions:

(a)  $f(x, y) = \frac{2xy}{1 + x^2y^2}$

(b)  $f(x, y) = \sqrt{2x - y^2}$

(c)  $f(x, y) = xe^{x\sqrt{y}}$

12. Find  $f_x$ ,  $f_y$ ,  $f_{xx}$ ,  $f_{xy}$ , and  $f_{yy}$  for the function  $f(x, y) = -4x^3y^5 + 9x^6y^2$ .
13. Find the local extrema for the function  $f(x, y) = x^3 - 3xy^2 + 6y^2$ .