MATH 142 - 502, 504

1. Find the domain of the function

$$f(x) = \begin{cases} x+4, & \text{if } x > 3\\ \frac{x}{\sqrt{x+2}}, & \text{if } x \le 3 \end{cases}$$

- 2. Which of the following are polynomials? For each polynomial determine its degree and circle its leading coefficient.
 - (a) $f(x) = x^5 + x^2 6 x^{11}$
 - (b) $g(x) = 5\sqrt{x} + 4x 2$
 - (c) $h(x) = 2x^4 x^2 + x$
 - (d) $r(x) = x^2 + x^{-2}$
- 3. Solve for x

$$7^{x^2} = 7^{2x+3}$$

- 4. Amanda would like to remodel her kitchen in 4 years. How much should she invest now at 7% compounded monthly to have \$15,000 four years from now? Round your answer to the nearest cent.
- 5. Solve for x

$$\log_5 x + \log_5(x+3) = \log_5 10$$

6. The financial department of a company that produces pens obtains the following pricedemand equation

$$p = 35 - 0.34x$$

where p is the wholesale price in dollars per pen at which x pens are sold. The fixed costs for the company is \$200 and the variable costs is \$4.50 per pen. How many pens should be produced by the company to maximize the profit? Round the answer to the nearest integer.

- 7. A company that produces and sells T-shirts established the price-demand function p(x) = 21 0.1x, where p(x) is the price per T-shirt at which x T-shirts can be sold. Suppose that the company must produce at least 50 shirts and its cost equation is C(x) = 2x + 250. How many T-shirts must the company produce to break-even? Round your answer to the nearest T-shirt.
- 8. The quantity demanded of a certain brand of computers is 300/wk when unit price is \$450. For each decrease in unit price of \$30, the quantity demanded increases by 100 units. The company will not supply any computers if the unit price is \$250 or lower. However, they will supply 375 computers if unit price is \$325. Find the equalibrium price and the equilibrium quantity.
- 9. Suppose \$29,000 is deposited into an account paying 7.5% annual interest. How much will be in the account after 5 years if the account is compounded continuously?

- 10. Express the given quatities as a single logarithm:
 - (a) $\log_2 x + 5\log_2(x+1) + \frac{1}{2}\log_2(x-1)$ (b) $2\ln 4 - \ln 2$
- 11. How long will it take for the amount in an account to triple if the money is compounded continuously at an annual interest rate of 4.7%?
- 12. Evaluate the limit

(a)
$$\lim_{x \to 5} \frac{x^2 - 25}{x^2 + x - 30}$$

(b) $\lim_{h \to 0} \frac{\sqrt{a + h} - \sqrt{a}}{h}, a \ge 0.$
(c) $\lim_{x \to \infty} \frac{2x^3 + 1}{4 - x - x^3}$

13. Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2 + 4}{3x^2 - 3}$.

14. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2 + 1 & , & \text{if } x < 2, \\ x + 2 & , & \text{if } x \ge 2. \end{cases}$$

- 15. The displacement of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$ (*t* is in seconds).
 - (a) Find the average velocity when t changes from t = 1 to t = 2.
 - (b) Find the velocity of the object when t = 1.
- 16. Find the equation of the tangent line to the curve $y = 2x^2 3$ at the point (2,5).
- 17. Let

$$p = 25 - 0.01x$$
 and $C(x) = 2x + 9000$

where $0 \le x \le 2500$, be the price-demand equation and cost function, respectively, for the manufacture of umbrellas.

- (a) Find the exact cost of producing the 31st umbrella.
- (b) Use the marginal cost to approximate the cost of producing the 31st umbrella.
- 18. Find the equation of the tangent line to the graph of the function $f(x) = \ln(1 x^2 + 2x^4)$ at the point where x = 1.
- 19. Find the value(s) of x where the tangent line to the graph of the function $y = 5e^{x^2-4x+1}$ is horizontal.
- 20. Find each derivative

(a)
$$\frac{d}{dx} \log_3(\sqrt[4]{4x^3 + 5x + 7})$$

(b) $\frac{d}{dx} 8^{1-2x^3}$
(c) $\frac{d}{dx} \frac{3x^2}{(x^2 + 5)^3}$
(d) $\frac{d}{dx} [(x^2 + x - 3)e^{2x+3}]$

21. Given the price-demand equation

$$0.02x + p = 60$$

- (a) Find the elasticity of demand E(p).
- (b) For which values of p is the revenue maximized? Find the maximum revenue.
- (c) If p = \$10 and the price is increased by 5%, what is the approximate change in demand?
- (d) If p = \$40 and the price is decreased, will revenue increase or decrease?
- 22. Find f''(x) for the functions

(a)
$$f(x) = x^2(2x^3 - 5)^4$$

(b) $f(x) = \frac{2}{x} - \frac{6}{x^3}$

23. Find the limit

(a)
$$\lim_{x \to \infty} (x - x^3)$$

(b) $\lim_{x \to -\infty} (x - x^3)$
(c) $\lim_{x \to -\infty} \frac{x^3 - 1}{2x^4 + 1}$
(d) $\lim_{x \to \infty} \frac{x^4 - 1}{1 - x^3}$
(e) $\lim_{x \to \infty} \frac{x^4 - 2x^2 + 3}{1 - x^4}$
(f) $\lim_{x \to -\infty} \frac{e^{-2x} + 4}{3 - 2e^{-2x}}$

24. Find the horizontal asymptotes of the function

(a)
$$f(x) = \frac{x^4 + 3x}{2 - 6x^4}$$

(b) $f(x) = \frac{x}{x^2 + 4}$

25. Given the graph of the function y = f(x).



- (a) Find the intervals on which f'(x) > 0.
- (b) Find the intervals on which f'(x) < 0.
- (c) Find x-coordinates of the points where f'(x) = 0.
- (d) Find the intervals on which f''(x) > 0.
- (e) Find the intervals on which f''(x) < 0.
- (f) Find x-coordinates of the points where f''(x) = 0.
- 26. Given the graph of the derivative f'(x) of the function f(x).



- (a) Find the intervals on which f(x) is increasing/decreasing.
- (b) Find x-coordinates of the critical values for f(x).
- (c) Find the intervals on which f(x) is convcave up/concave down.
- (d) Find *x*-coordinates of the inflection points.

- 27. Given the function $f(x) = \frac{1}{4}x^4 4x$.
 - (a) Find critical values of f(x).
 - (b) Find intervals on which f(x) is increasing and decreasing.
 - (c) Find local extrema for f(x).
 - (d) Find intervals on which f(x) is concave upward and concave downward.
 - (e) Find all inflection points of f(x).
- 28. Find the absolute maximum and absolute minimum for the function $f(x) = \sqrt{9 x^2}$ on the interval [-1, 2].
- 29. Find the absolute maximum and minimum for the function $f(x) = \frac{x^2 1}{x^2 + 1}$.
- 30. A box with a square base and open top must have a volume of 32000 cm³. Find the dimensions of the box that minimize the amount of material used.
- 31. Find the following indefinite integrals:

(a)
$$\int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$$

(b)
$$\int \frac{x^3 - 3}{x^4 - 12x + 3} dx$$

(c)
$$\int \frac{(\ln x)^2}{x} dx$$

(d)
$$\int \frac{x}{(5 - 2x^2)^5} dx$$

(e)
$$\int \frac{x}{\sqrt{3 - x}} dx$$

(f)
$$\int e^{2x - 1} dx$$

32. The weekly marginal revenue from the sale of x pairs of tennis shoes is given by

$$R'(x) = 40 - 0.002x + \frac{200}{x+1}, \quad R(0) = 0,$$

where R(x) is revenue in dollars. Find the revenue function R(x). Find the revenue from the sale of 1000 pairs of shoes.

- 33. Use L_6 and R_6 to approximate $\int_{2}^{5} (0.25x^2 4) dx$.
- 34. Evaluate each of the definite integrals:

(a)
$$\int_{0}^{A} 32(x^{2}+1)^{7}x \, dx \quad (A>0)$$

(b)
$$\int_{B}^{2} \left(5x - 4\frac{x^2}{\sqrt[4]{x^3}} \right) dx, \quad (0 < B < 2)$$

35. Find the average value of the function $f(x) = 4x - 3x^2$ over the interval [-2, 2].

36. Find the area of the region bounded by:

- (a) $y = 3 x^2$, $y = 2x^2 4x$ (b) $y = x^3$, y = 4x(c) $y = -x^2 - 2x$, y = 0, x = -2, x = 1.
- 37. Find the consumers' surplus and producers' surplus at the equilibrium price level for the given price-demand and price-supply equations.

$$p = D(x) = 70 - 0.2x$$

$$p = S(x) = 13 + 0.0012x^{2}$$

Round all values to the nearest integer.

- 38. Let f(x,y) = 2x 3y + 14 and $g(x,y) = \frac{10}{x^2 + 4y}$. Find f(2,-3) 4g(-1,2).
- 39. Weston Publishing publishes a deluxe edition and a standard edition of its English language dictionary. Weston's management estimates that the number of deluxe editions demanded is x copies/day and the number of standard editions demanded is y copies/day when the unit prices are

$$p = 20 - 0.005x - 0.001y$$
$$q = 15 - 0.001x - 0.003y$$

dollars, respectively. Find the daily total revenue function R(x, y). Evaluate R(7, 3).

40. Find f_x and f_y for the functions:

(a)
$$f(x,y) = \frac{2xy}{1+x^2y^2}$$

(b) $f(x,y) = \sqrt{2x-y^2}$
(c) $f(x,y) = xe^{x\sqrt{y}}$

41. Find f_x , f_y , f_{xx} , f_{xy} , and f_{yy} for the function $f(x, y) = -4x^3y^5 + 9x^6y^2$.

42. Find the local extrema for the function $f(x, y) = x^3 - 3xy^2 + 6y^2$.