42. Find the local extrema for the function $f(x, y) = x^3 - 3xy^2 + 6y^2$.

$f_x = 3x^2 - 3y^2 = 0$

$f_y = -6xy + 12y = 0$

$\begin{cases} \frac{3x^2 - 3y^2}{3} = \frac{0}{3} \\ \frac{-6xy + 12y}{6} = \frac{0}{6} \end{cases}$

$\begin{cases} x^2 - y^2 = 0 \\ -xy + 2y = 0 \end{cases} \Rightarrow \quad x^2 = y^2, \quad x = y$ or $x = -y$

$x = y$ plug into the 2nd equation:

$-y^2 + 2y = 0$

$-y(y-2) = 0$

$y_1 = 0, \quad y_2 = 2$

$x = y \Rightarrow x_1 = 0$

$x_2 = 2$

$\boxed{(0,0), (2,2)}$

$x = -y$ and plug it into the 2nd equation:

$-(-y)y + 2y = 0$

$y^2 + 2y = 0$

$y(y+2) = 0$

$y = 0, \quad y = -2$

$x = -y$

$y = 0 \Rightarrow x = 0$

$y = -2 \Rightarrow x = 2$

$\boxed{(0,0), (2,-2)}$

$(0,0), (2,2), (2,-2)$ critical points.

$f_{xx} = (3x^2 - 3y^2)'_x = 6x$

$f_{xy} = (3x^2 - 3y^2)'_y = -6y$

$f_{yy} = (-6xy + 12y)'_y = -6x + 12$

(0,0) test fails.

$f_{xx} = 0 = A$

$f_{xy} = 0 = B$

$f_{yy} = 12 = C$

$B^2 - AC = 0$

(2,2) saddle point

$f_{xx}(2,2) = 12 > 0 \quad = A$

$f_{xy}(2,2) = -12 \quad = B$

$f_{yy}(2,2) = 0 \quad = C$

$AC - B^2 = 0(12) - (-12)^2 = -144 < 0$

(2,-2) saddle point

$f_{xx}(2,-2) = 12 = A$

$f_{xy}(2,-2) = 12 = B$

$f_{yy}(2,-2) = 0 = C$

$AC - B^2 = -(12)^2 = -144 < 0$

21. Given the price-demand equation

$$0.02x + p = 60$$

(a) Find the elasticity of demand $E(p)$.

(b) For which values of p is the revenue maximized? Find the maximum revenue.

(c) If $p = \$10$ and the price is increased by 5%, what is the approximate change in demand?

(d) If $p = \$40$ and the price is decreased, will revenue increase or decrease?

$$\boxed{E(p) = -\frac{pf'(p)}{f(p)}}$$

$0.02x+p=60$ solve for x

$0.02x = 60-p$

$x = \frac{60}{0.02} - \frac{1}{0.02}p$

$\boxed{x = 3000 - 50p}$ $\boxed{3000-50p = f(p)}$

$E(p) = -\frac{p(3000-50p)'}{3000-50p} = \frac{-p(-50)}{3000-50p} = \frac{50p}{3000-50p}$

$$\boxed{E(p) = \frac{p}{60-p}}$$

(b) (1) The revenue is maximized when $E(p)=1$

$\frac{p}{60-p} = 1$

$p = 60-p$

$2p = 60$

$\boxed{p = \$30}$

$R(p) = pf(p) = p(3000-50p)$

$R(30) = 30(3000-50(30)) = \boxed{\$45000}$

(2) $0.02x+p=60$, solve for p

$p = 60-0.02x$ – price-demand equation.

The revenue $R(x) = xp(x)$

$= x(60-0.02x)$

$R(x) = 60x - 0.02x^2$

to be maximized

$R'(x) = 60 - 0.04x = 0$

$0.04x = 60$

$x = \frac{60}{0.04} = 1500$

$p(1500) = 60 - 0.02(1500)$

$= \boxed{\$30}$

max Revenue $R(1500) = (1500)(30) = \boxed{\$45000}$

(c) $p = \$10$, increased by 5%.

$E(10) = \frac{10}{60-10} = \frac{10}{50} = 0.2 < 1$ inelastic

(change in demand) $=$ $E(10)$ (change in price)

$= 0.2(5\%)$

$= 1\%$

$\boxed{\text{demand will } \underline{\text{decrease}} \text{ by } 1\%}$

$\boxed{\text{Revenue will } \underline{\text{increase}}}$

(d) $p=40$, price is decreased.

$E(40) = \frac{40}{60-40} = 2 > 1$ elastic

$\boxed{\text{The revenue will increase}}$

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