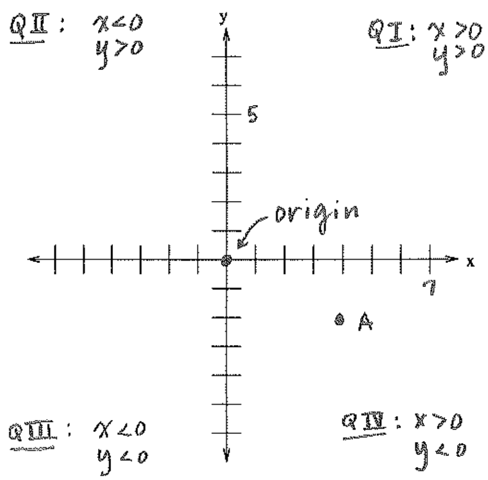
Prerequisite Information for Math 142

Functions

Cartesian Coordinate System



There are four quadrants: I – IV

Ordered Pairs: (x, y)

- Origin: $(0,0)$
- Point A = ? ⇒ $(4,-2)$

✻ It is important to always indicate your scale for both x and y values on your graphs.

Functions

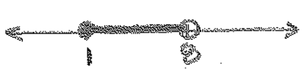
Def: A **function** is a rule (process) that assigns to each element in the *domain* (the set of independent variables, x) ONE AND ONLY ONE element in the *range* (the set of dependent variables).

Interval Notation

[ ]: Square brackets = INCLUDE ENDPOINTS ( ): open brackets = DON'T INCLUDE ENDPOINTS

Ex: Draw number lines and inequalities that represent the following domains.

(a) $[1,8)$



$1 \le x < 8$

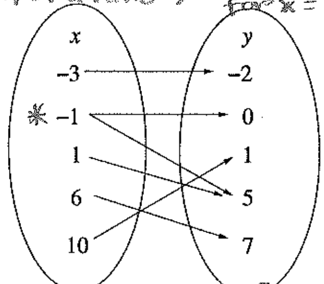
(b) $(-4,3) \cup (4,\infty)$

$-4 < x < 3$ or $x > 4$

✻ Always use OPEN brackets with ∞ and $-\infty$.

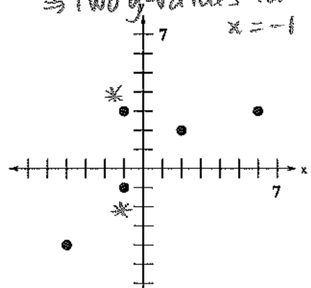
Ex: Find the domain and range of each correspondence below and then determine whether each is a function of x.

✻ NOT a func ⇒ Two y-values for $x=-1$



D: $\{-3,-1,1,6,10\}$
R: $\{-2,0,1,5,7\}$

✻ NOT a func ⇒ Two y-values for $x=-1$



D: $\{-4,-1,2,6\}$
R: $\{-4,-1,2,3\}$

FUNCTION

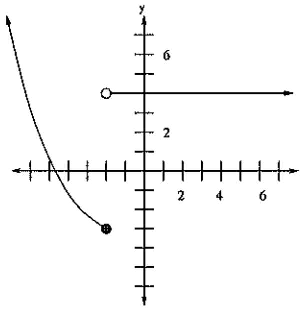
$2x + 4y = 8$

Solve for y: $4y = -2x + 8$
$y = -\frac{1}{2}x + 2$

- Any x can be input ⇒ D: $(-\infty,\infty) = \mathbb{R}$
- Every input x results in only one y, and all y-values are obtained ⇒ R: $(-\infty,\infty) = \mathbb{R}$

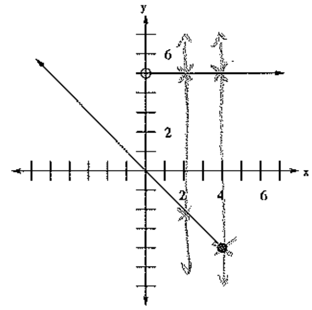
Vertical Line Test: If *any* vertical line passes through two or more points on the graph of an equation, then the equation does *not* define a function. * Look at the last example on the last page *

Ex: Find the domain and range of each correspondence below and then determine whether each is a function of x.



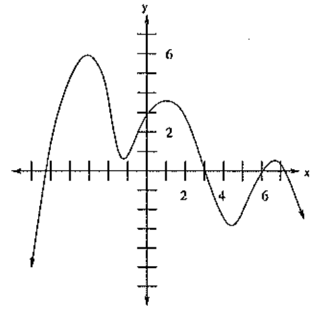
D: $(-\infty, \infty)$
R: $[-3, \infty)$

FUNCTION



D: $(-\infty, \infty)$
R: $[-4, \infty)$

NOT A FUNCTION



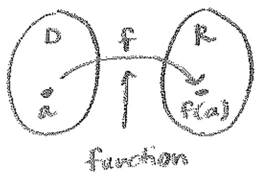
D: $(-\infty, \infty)$
R: $(-\infty, 6]$

FUNCTION

Function Notation

If a correspondence is a function, we use the notation $f(x)$, read "f of x" or "f at x". Ordered pairs of the form (x, y) can then be written as $(x, f(x))$.

↳ * NOT f times x !! *



$\Rightarrow (a, f(a))$

If $x = b$ is not in the domain of $f(x)$, then we say $f(x)$ is undefined at $x = b$ OR $f(x)$ does not exist (DNE) at $x = b$.

Ex: Let $k(x) = x^2 + 2x + 1$ and $g(x) = \sqrt{x+5}$. Find the following:

(a) $k(-2) = (-2)^2 + 2(-2) + 1 = 4 - 4 + 1 = \boxed{1}$ $\Rightarrow$ meaning $(-2, 1)$ is a point on the graph of $k(x)$

(b) $g(1) = \sqrt{1+5} = \boxed{\sqrt{6}}$ $\Rightarrow$ $(1, \sqrt{6})$ is on the graph of $g(x)$

(c) $k(a+b) = (a+b)^2 + 2(a+b) + 1 = (a+b)(a+b) + 2a + 2b + 1$
$= a^2 + ab + ba + b^2 + 2a + 2b + 1$
$= \boxed{a^2 + 2ab + b^2 + 2a + 2b + 1}$

(d) $k(x+h) = (x+h)^2 + 2(x+h) + 1$
$= \boxed{x^2 + 2xh + h^2 + 2x + 2h + 1}$

* NOTE: $(a+b)^2 \neq a^2 + b^2$

* NOTE: $k(x+h) \neq k(x) + h$

* This concept will be used a lot throughout the semester.

Difference Quotient: $\dfrac{f(x+h)-f(x)}{h}$

(e) $\dfrac{k(x+h)-k(x)}{h}$ (Recall: $k(x) = x^2+2x+1$)

$k(x+h) = (x+h)^2+2(x+h)+1 = x^2+2xh+h^2+2x+2h+1$

$k(x+h)-k(x) = (\cancel{x^2}+2xh+h^2+\cancel{2x}+2h+\cancel{1}) - (\cancel{x^2}+\cancel{2x}+\cancel{1})$

$= 2xh+h^2+2h$

$$\frac{k(x+h)-k(x)}{h} = \frac{2xh+h^2+2h}{h} = \frac{\cancel{h}(2x+h+2)}{\cancel{h}} = \boxed{2x+h+2}$$

Evaluating Functions on the Calculator

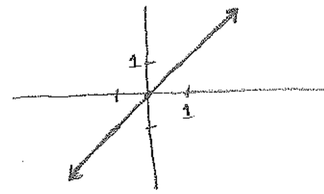
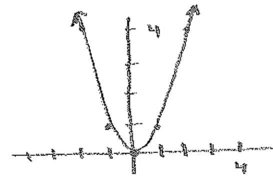
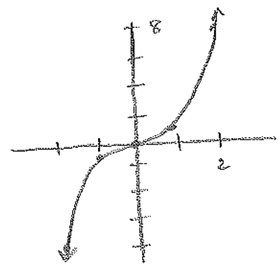
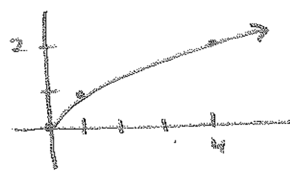
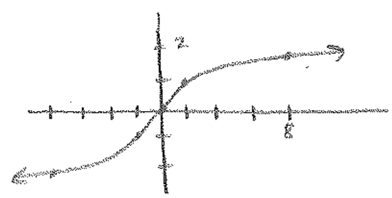
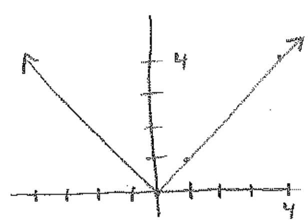
→ Only used when given specific numerical values of x.

1. Press $\boxed{Y=}$ and type your function into any y value, say Y_1. (Use the $\boxed{X,T,\theta,n}$ button for entering x.)
2. Quit to your home screen by pressing $\boxed{2^{nd}}$ $\boxed{\text{MODE}}$.
3. Press $\boxed{\text{VARS}}$. Go right to *Y-VARS*. Select 1:*Function* and then select the name of where your function is stored, say 1:Y_1.
4. Finish the function notation by pressing the left parenthesis key, x value, right parenthesis key and $\boxed{\text{ENTER}}$.

* Practice using $k(x)$ and $g(x)$ from the last page. Find $k(-2)$ and $g(1)$ and compare to the values we obtained by hand.
(NOTE: $\sqrt{6}$ is an EXACT value.)

Basic Elementary Functions (Parent Functions)

* You should know these without your calculator.

Name	Equation	Graph ($\overset{x}{\text{Domain}}$ and $\overset{y}{\text{Range}}$)	
Identity Function:	$f(x) = x$		D: $\mathbb{R}$ R: $\mathbb{R}$
Square Function:	$h(x) = x^2$		D: $\mathbb{R}$ R: $[0, \infty)$
Cube Function:	$m(x) = x^3$		D: $\mathbb{R}$ R: $\mathbb{R}$
Square Root Function: An example of an <u>even</u> root	$n(x) = \sqrt{x}$		D: $[0, \infty)$ R: $[0, \infty)$
Cube Root Function: An example of an <u>odd</u> root	$p(x) = \sqrt[3]{x}$		D: $\mathbb{R}$ R: $\mathbb{R}$
Absolute Value Function:	$g(x) = \lvert x \rvert$		D: $\mathbb{R}$ R: $[0, \infty)$

Note: The **absolute value function** $f(x) = |x|$ is defined by

$$f(x) = \begin{cases} -x & , \overset{\text{Domain}}{x < 0} \\ x & , x \geq 0 \end{cases}$$

* Different rules for different values of x.

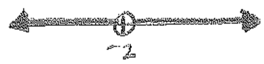
Finding the Domain of a Function Algebraically
Find all of the values of the independent variable (x) that produce *real* values for the dependent variable (y).

Ex: Find the domain of the following functions. Write your answer in interval notation.

Polynomial (a) $g(x) = x^8 + 10x$ $\boxed{D: (-\infty, \infty)}$

Rational Function (b) $f(x) = \dfrac{x^2-4}{x+2}$

$x+2 \neq 0$
$x \neq -2$
you can't divide by zero.



$\boxed{D: (-\infty, -2) \cup (-2, \infty)}$

Even Root (c) $k(x) = \sqrt{7-x}$

$7 - x \geq 0$
$-x \geq -7$
$x \leq 7$

you can't take an even root of a negative #.

$\boxed{D: (-\infty, 7]}$

Even Root and Denominator (d) $r(x) = \dfrac{\sqrt{x+3}}{x-1}$

(even root)
$x + 3 \geq 0$
$x \geq -3$

(denom.)
$x - 1 \neq 0$
$x \neq 1$

BOTH must be TRUE.

$\boxed{D: [-3, 1) \cup (1, \infty)}$

Denominator and Even Root in Denom. (e) $f(x) = \dfrac{x^2+4x+1}{x\sqrt{x-6}}$

(denom)
$x \neq 0$

(even root in denom)
$x - 6 > 0$
$x > 6$

$\boxed{D: (6, \infty)}$

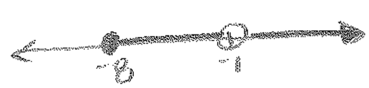
Odd Root (f) $h(x) = \sqrt[3]{x+4}$ $\boxed{D: (-\infty, \infty)}$

you can take an odd root of any real # $\Rightarrow$ $(-2)^3 = -8$ so... $-2 = \sqrt[3]{-8}$

Even Root and Odd Root in Denom. (g) $g(x) = \dfrac{\sqrt{x+8}}{\sqrt[5]{x+1}}$

(even root in num)
$x + 8 \geq 0$
$x \geq -8$

(odd root in denom)
$x + 1 \neq 0$
$x \neq -1$



$\boxed{D: [-8, -1) \cup (-1, \infty)}$

Some Applications of Functions

Linear Cost Function: $C(x) = cx + F$

x = # of items made
c = cost/item
F = fixed costs.

Price-Demand Function: $p(x) = mx + b$

x = # of items sold at \$p/item
m should be negative

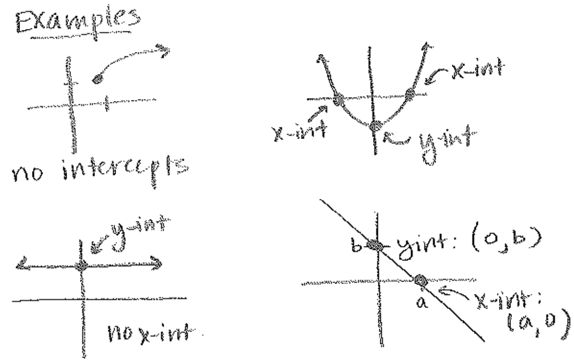
Revenue Function: $R(x) =$(price)(quantity) *

= (price-demand)(quantity sold) when price is not constant
$= (mx+b)(x)$
$= mx^2 + bx$

Profit Function: $P(x) = R(x) - C(x)$ (functions from above)
= \$ in − \$ out

Linear Functions and Straight Lines

Intercepts



x-intercept: where a graph crosses the x axis ($\Longrightarrow y = 0$)
y-intercept: where a graph crosses the y axis ($\Longrightarrow x = 0$)

* Graphs of functions will have at most one y-int. If x=0 is not in the domain, then there will be no y-int.

Linear Functions & Equations

Def: A function f is a **linear function** if

$$f(x) = mx + b \qquad m \neq 0$$

where m and b are real numbers.

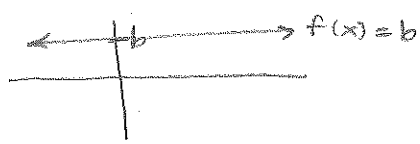
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Def: If $m = 0$, then f is called a **constant function**, $f(x) = b$.

Domain: $\mathbb{R}$

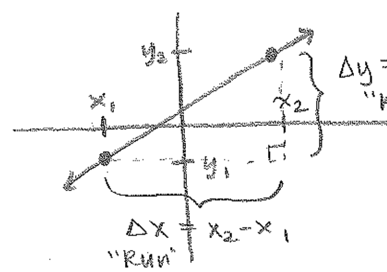
Range: $\{b\}$



Def: The **slope** of a line, denoted by m, measures the "steepness" of the line relative to the x-axis. Given two points on a line (x_1, y_1) and (x_2, y_2), the slope of the line is computed by

* Similar Δ's show any two points on the line will work.
Two pts uniquely determine a line.

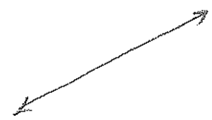
$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1 \neq x_2)$$
$$= \frac{\Delta y}{\Delta x}$$
$$= \frac{\text{"rise"}}{\text{"run"}}$$



Ex: Find the slope of the line passing through the points $(1, -2)$ and $(-4, 8)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{8-(-2)}{-4-1} = \frac{10}{-5} = \boxed{-2}$$

Line Properties

Type	*Slope*	*Example*
Horizontal	$m = 0$	
Vertical	m undefined	
Rising (Increasing)	$m > 0$	
Falling (Decreasing)	$m < 0$	

⇒ Vertical lines are NOT functions.

As x moves left to right.

Equations of Lines

Standard Form:	$Ax + By = C$	A and B not both 0.
Slope-Intercept Form:	$f(x) = y = mx + b$	Line with slope, m, & y-int at $(0, b)$.
Point-Slope Form:	$y - y_1 = m(x - x_1)$	Line with slope, m & thru pt (x_1, y_1).
Vertical Line:	$x = a$	a is a constant value; m undefined
Horizontal Line:	$y = b$	b is a constant value; $m = 0$

Ex: Find the x- and y-intercepts of $y = \frac{4}{7}(x - 7) + 1$.

x-int : $(y=0)$

$0 = \frac{4}{7}(x-7)+1$

$0 = \frac{4}{7}x - 4 + 1$

$0 = \frac{4}{7}x - 3$

$3 = \frac{4}{7}x$

$\left(\frac{7}{4}\right)3 = x$

$x = \frac{21}{4}$

$\boxed{\text{x-int: } \left(\frac{21}{4}, 0\right)}$

y-int : $(x=0)$

$y = \frac{4}{7}(0-7)+1$

$= \frac{4}{7}(-7)+1$

$= -4+1$

$y = -3$

$\boxed{\text{y-int: } (0, -3)}$

* you could plot these two pts and connect them if you were asked to graph the linear function.

Ex: Write an equation for each line described.

(a) The line passing through $(1, 2)$ and $(-4, 3)$.

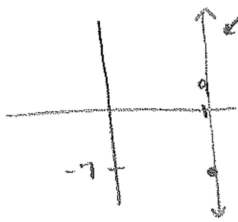
$m = \frac{\Delta y}{\Delta x} = \frac{3-2}{-4-1} = \frac{1}{-5}$

Pt-Slope Form $y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -\frac{1}{5}(x-1)}$

pt: $(1,2)$

(b) The vertical line passing through $(9, -7)$.



All pts on this line have the same x-value. $\Rightarrow$ $\boxed{x = 9}$

(c) The line crossing the $\underbrace{x\text{-axis at } x = 4}_{(4,0)}$ and the $\underbrace{y\text{-axis at } y = 5}_{(0,5)}$. ← y-int.

$m = \frac{\Delta y}{\Delta x} = \frac{5-0}{0-4} = \frac{-5}{4}$

Slope-Int. Form: $y = mx + b$

$\boxed{y = \frac{-5}{4}x + 5}$

(d) A linear function, $f(x)$, in which $\underbrace{f(0) = 8}_{(0,8)\ \text{y-int}}$ and $\underbrace{\text{slope is } 1/2}_{m = 1/2}$.

Slope-Int. Form: $\boxed{f(x) = \frac{1}{2}x + 8}$

Applications of Linear Functions

Linear Cost Function: The total cost of producing x items - taking into account both variable and fixed costs.

costs that exist even when no items are produced

$$C(x) = cx + F$$

fixed costs

cost/unit

points on the graph of the form $(x, C(x))$

Ex: KB & Co. is manufacturing insulated mugs. The company has monthly fixed costs of \$1500 and there is a total monthly cost of \$1800 when producing 100 mugs. Find the linear cost function for KB & Co.

Pts: $(0, 1500)$

$(100, 1800)$

$$c = m = \frac{\Delta C}{\Delta x} = \frac{1800-1500}{100-0} = \frac{300}{100} = 3$$

$$\boxed{C(x) = 3x + 1500}$$ where $\underbrace{x = \text{\# of mugs produced}}$

* define variables used if not given in the problem.

Linear Supply and Demand:

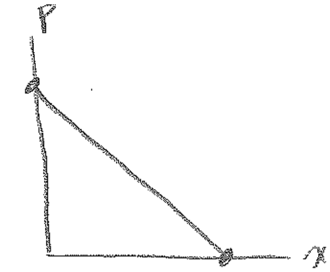
** points for linear supply and demand are always of the form

$(x, p) = (\text{quantity}, \text{price})$

D: $p = mx + b$

S: $p = mx + b$

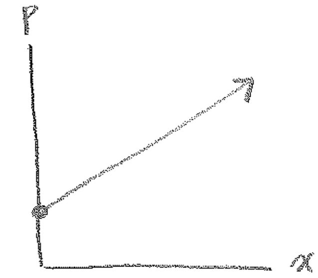
Price-Demand



From "consumer" standpoint

$\Rightarrow$ negative slope!

Price-Supply



From "producer" standpoint

$\Rightarrow$ positive slope!

Ex: At a price of \$100 per calculator, the annual supply and demand for calculators in a particular market are 1200 and 900, respectively. When the price rises to \$110, the supply increases to 1500 calculators while the demand decreases to 800 calculators. Assuming that the price-supply and the price-demand equations are linear, find the equations for each.

Supply: $(1200, 100)$
$(1500, 110)$

$$m = \frac{\Delta p}{\Delta x} = \frac{110-100}{1500-1200} = \frac{10}{300} = \frac{1}{30}$$

$$p - 100 = \frac{1}{30}(x - 1200)$$
$$= \frac{1}{30}x - 40$$

$$\boxed{S: \quad p = \frac{1}{30}x + 60}$$

Demand: $(900, 100)$
$(800, 110)$

$$m = \frac{\Delta p}{\Delta x} = \frac{110-100}{800-900} = \frac{10}{-100} = \frac{-1}{10}$$

$$p - 100 = \frac{-1}{10}(x - 900)$$
$$= \frac{-1}{10}x + 90$$

$$\boxed{D: \quad p = \frac{-1}{10}x + 190}$$

Quadratic Functions

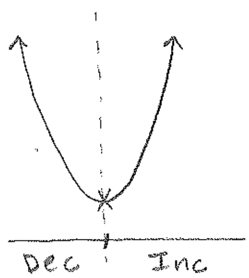
Properties of Quadratic Functions

Def: A function of the form

$$f(x) = ax^2 + bx + c$$

is called a **quadratic function**, where a, b, and c are real numbers and $a \neq 0$.

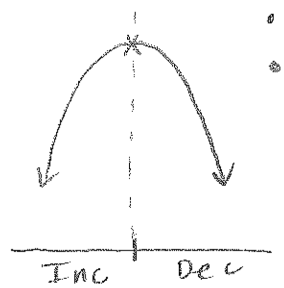
$a > 0$



- opens upward
- vertex is a MIN

D: ℝ
R: [?, ∞)
y-value of vertex

$a < 0$



- opens downward
- vertex is a MAX

D: ℝ
R: (−∞, ?]
y-value of vertex

Def: The maximum or minimum of a quadratic function $f(x)$, whose graph is known as a *parabola*, is called its **vertex** and has coordinates

$$(x, y) = (h, k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

* The line of symmetry thru the vertex is called the axis of the parabola. $(x = h)$

Ex: Given $f(x) = \overset{a}{-2}x^2 \overset{b}{\boxed{+4}}x \overset{c}{+3}$, determine

(a) the direction the parabola opens. $\boxed{\text{downward}}$ $(a < 0)$

(b) the y-intercept. $(x = 0)$ $f(0) = -2(0)^2 + 4(0) + 3 = 3 \Rightarrow \boxed{(0, 3)}$

(c) the vertex (is it a max or min?).

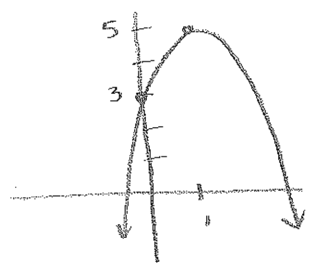
$h = \dfrac{-b}{2a} = \dfrac{-4}{2(-2)} = 1$

$k = f(1) = (-2)(1)^2 + 4(1) + 3 = 5$

$\boxed{\text{vertex: } (1, 5) \text{ is a max}}$

(d) the domain of $f(x)$. $\boxed{\mathbb{R}}$

(e) the range of $f(x)$. $\boxed{(-\infty, 5]}$



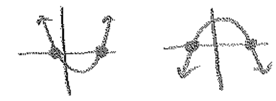
(f) the interval(s) where $f(x)$ is increasing/decreasing.

Inc: $(-\infty, 1)$
Dec: $(1, \infty)$

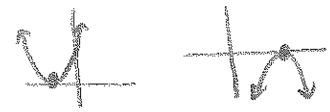
* you are giving intervals of x-values

Determining Zeros of a Quadratic Function

Ex: Two Real Roots



One Real Root



No Real Roots

Def: The real **zeros** (or **roots**) of a function are its $\underbrace{x\text{-intercepts}}_{y=0}$.

$$ax^2+bx+c=0$$

To find the EXACT zeros of a *quadratic* function:

1. Factor, set each factor equal to zero, and solve for x.

2. Use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

To approximate the zeros of any function, you can use your calculator:

1. Graph your function on a window where the zeros are visible.
2. Press $\boxed{2^{nd}}$ $\boxed{\text{TRACE}}$ and select option 2:*zero.*
3. Follow the directions on the screen, using your left and right arrow keys to move along the curve.

Ex: Find the EXACT zeros of the following functions:

(a) $f(x) = 6x^2 - 7x - 3$

$= (3x+1)(2x-3) = 0$

$\Rightarrow 3x+1=0 \qquad 2x-3=0$

$3x=-1 \qquad 2x=3$

$x=-1/3 \qquad x=3/2$

$\Rightarrow \boxed{\text{zeros @ } x=\frac{-1}{3} \text{ and } x=\frac{3}{2}}$

(b) $g(x) = -2x^2 + 4x + 3$

$$x = \frac{-4 \pm \sqrt{4^2-4(-2)(3)}}{2(-2)} = \frac{-4 \pm \sqrt{40}}{-4}$$

$\Rightarrow \boxed{\text{zeros @ } x=\frac{-4+\sqrt{40}}{-4} \text{ and } x=\frac{-4-\sqrt{40}}{-4}}$

Ex: Solve $x^2 + 3x = 40$

$x^2+3x-40=0$

$(x+8)(x-5)=0$

$\Rightarrow x+8=0 \qquad x-5=0$

$\boxed{x=-8} \qquad \boxed{x=5}$