MATH 142 – 502, 504 **Practice Problems for Test 1** **Fall 2014**

1. Find the domain of the function

$$f(x) = \begin{cases} x+4, & \text{if } x > 3 \\ \dfrac{x}{\sqrt{x+2}}, & \text{if } x \leq 3 \end{cases}$$

2. Which of the following are polynomials? For each polynomial determine its degree and circle its leading coefficient.

   (a) $f(x) = x^5 + x^2 - 6 - x^{11}$ polynomial of 11 th degree

   (b) $g(x) = 5\sqrt{x} + 4x - 2$ not a polynomial

   (c) $h(x) = 2x^4 - x^2 + x$ polynomial of 4 th degree

   (d) $r(x) = x^2 + x^{-2}$ not a polynomial

3. Solve for x

$$7^{x^2} = 7^{2x+3}$$

4. Amanda would like to remodel her kitchen in 4 years. How much should she invest now at 7% compounded monthly to have \$15,000 four years from now? Round your answer to the nearest cent.

5. Solve for x

$$\log_5 x + \log_5(x+3) = \log_5 10$$

~~6. Write as a single logarithm~~

7. The financial department of a company that produces pens obtains the following price-demand equation

$$p = 35 - 0.34x$$

   where p is the wholesale price in dollars per pen at which x pens are sold. The fixed costs for the company is \$200 and the variable costs is \$4.50 per pen. How many pens should be produced by the company to maximize the profit? Round the answer to the nearest integer.

8. A company that produces and sells T-shirts established the price-demand function $p(x) = 21 - 0.1x$, where $p(x)$ is the price per T-shirt at which x T-shirts can be sold. Suppose that the company must produce at least 50 shirts and its cost equation is $C(x) = 2x + 250$. How many T-shirts must the company produce to break-even? Round your answer to the nearest T-shirt.

9. The quantity demanded of a certain brand of computers is 300/wk when unit price is \$450. For each decrease in unit price of \$30, the quantity demanded increases by 100 units. The company will not supply any computers if the unit price is \$250 or lower. However, they will supply 375 computers if unit price is \$325. Find the equaibrium price and the equilibrium quantity.

10. Suppose \$29,000 is deposited into an account paying 7.5% annual interest. How much will be in the account after 5 years if the account is compounded continuously?

11. Express the given quatities as a single logarithm:

    (a) $\log_2 x + 5\log_2(x+1) + \frac{1}{2}\log_2(x-1)$

    (b) $2\ln 4 - \ln 2$

12. How long will it take for the amount in an account to triple if the money is compounded continuously at an annual interest rate of 4.7%?

13. Evaluate the limit

    (a) $\lim_{x\to 5} \frac{x^2-25}{x^2+x-30}$

    (b) $\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}$, $a \geq 0$.

    (c) $\lim_{x\to\infty} \frac{2x^3+1}{4-x-x^3}$

14. Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2+4}{3x^2-3}$.

15. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2+1 & , \text{ if } x < 2, \\ x+2 & , \text{ if } x \geq 2. \end{cases}$$

16. The displacement of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$ (t is in seconds).

    (a) Find the average velocity when t changes from $t = 1$ to $t = 2$.

    (b) Find the velocity of the object when $t = 1$.

17. Find the equation of the tangent line to the curve $y = 2x^2 - 3$ at the point (2,5).

#1.

$$f(x) = \begin{cases} x+4, & x>3 \\ \frac{x}{\sqrt{x+2}}, & x \leq 3 \end{cases}$$

$x+4$ is defined for all x.

$\frac{x}{\sqrt{x+2}}$ is defined if $x+2>0$ or $x>-2$

domain: $(-2,3] \cup (3,\infty) = \boxed{(-2,\infty)}$

#3.

$7^{x^2} = 7^{2x+3}$

$x^2 = 2x+3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x_1 = 3, \ x_2 = -1}$

#4. Use TVM solver for

$N = 4 \cdot 12$

$I\% = 7$

$PV = 0$

$PMT = 0$

$FV = 15000$

$P/Y = 12$

$C/Y = 12$

and then solve for PV: $\boxed{\$11345.98}$

#5. $\log_5 x + \log_5 (x+3) = \log_5 10$

$\log_5 x(x+3) = \log_5 10$

$x(x+3) = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\cancel{x_1 = -5}$, $\boxed{x_2 = 2}$

$x_1 < 0$ - not valid

Domain:

$x > 0 \Rightarrow x > 0$

$x + 3 > 0 \quad x > -3$

intersection: $\boxed{x > 0}$

#7. $p = 35 - 0.34x$

$C(x) = 4.5x + 200$

$R(x) = xp(x) = (35 - 0.34x)x = 35x - 0.34x^2$

Profit: $P(x) = R(x) - C(x) = 35x - .34x^2 - 4.5x - 200$

$= \underbrace{-.34}_{a}x^2 + \underbrace{30.5}_{b}x - 200$

$x_{max} = -\frac{b}{2a}$

$= -\frac{30.5}{2(-0.34)} \approx \boxed{45}$

#8. $p(x) = 21 - .1x$

$C(x) = 2x + 250, \ x \geq 50$

$R(x) = xp(x) = 21x - .1x^2$

Break even when $R(x) = C(x)$:

$2x + 250 = 21x - .1x^2$

Graph $Y_1 = 2x + 250$ on the same screen

$Y_2 = 21x - .1x^2$

then do $\boxed{\text{2nd}}$ $\boxed{\text{trace}}$ then $\boxed{\text{5: intersect}}$ to find the points. Choose one that $y \geq 50$.

$$\boxed{x = 176}$$

#9. <u>demand</u>: $p - p_0 = m(x - x_0)$

<u>passes</u> through $(\underbrace{300}_{x_0}, \overbrace{450}^{p_0})$

$$m = \frac{\Delta p}{\Delta x} = \frac{-30}{100} = -.3$$

Equation: $p - 450 = -.3(x - 300)$

$$p = -0.3x - .3(-300) + 450$$

$$\boxed{p = -.3x + 540}$$

<u>Supply</u>: passes through $(\overbrace{0}^{x_0}, \overbrace{250}^{p_0})$

$(375, 325)$

$$p - p_0 = m(x - x_0)$$

$$m = \frac{\Delta p}{\Delta x} = \frac{325 - 250}{375} = .2$$

$$p - 250 = 0.2x$$

$$p = 0.2x + 250$$

<u>Equilibrium market</u>: supply = demand

$$-0.3x + 540 = 0.2x + 250$$

$$0.5x = 290$$

$$x = \frac{290}{0.5} = \boxed{580 \text{ equilibrium quantity}}$$

$$p = 0.2(580) + 250 = \boxed{\$366 \text{ equilibrium price}}$$

#10. $A = Pe^{rt}$

$P = 29,000$

$r = 7.5\% = 0.075$

$t = 5$

$$A = 29000\,e^{(0.075)(5)} \approx \boxed{42194.75}$$

#11. (a) $\log_2 x + 5\log_2(x+1) + \frac{1}{2}\log_2(x-1)$

$= \log_2 x + \log_2(x+1)^5 + \log_2(x-1)^{1/2}$

$= \boxed{\log_2\left(x(x+1)^5(x-1)^{1/2}\right)}$

(b) $2\ln 4 - \ln 2 = \ln(4^2) - \ln 2 = \ln\frac{4^2}{2} = \boxed{\ln 8}$

#12. $r = 4.7\% = 0.047$

$A = 3P$

$A = Pe^{rt}$

$3\cancel{P} = \cancel{P}e^{rt}$

$3 = e^{rt}$

$\ln 3 = \ln(e^{rt})$

$\ln 3 = rt\,\cancel{\ln e}$ 1

$rt = \ln 3$

$t = \frac{\ln 3}{t} = \frac{\ln 3}{0.047} \approx \boxed{23\text{(years)}}$

#13. (a) $\lim_{x\to5} \frac{x^2-25}{x^2+x-30} = \left|\frac{0}{0}\right| = \lim_{x\to5}\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} = \lim_{x\to5}\frac{x+5}{x+6} = \boxed{\frac{10}{11}}$

(b) $\lim_{h\to0}\frac{\sqrt{a+h}-\sqrt{a}}{h} = \lim_{h\to0}\frac{(\sqrt{a+h}-\sqrt{a})(\sqrt{a+h}+\sqrt{a})}{h(\sqrt{a+h}+\sqrt{a})}$

$= \lim_{h\to0}\frac{(\sqrt{a+h})^2-(\sqrt{a})^2}{h(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to0}\frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to0}\frac{\cancel{h}}{\cancel{h}(\sqrt{a+h}+\sqrt{a})}$

$= \lim_{h\to0}\frac{1}{\sqrt{a+h}+\sqrt{a}} = \boxed{\frac{1}{2\sqrt{a}}}$

(c) $\lim_{x\to\infty}\frac{\boxed{2}x^3+1}{4-x-\boxed{1}x^3} = \frac{2}{-1} = \boxed{-2}$ same

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#14. $y = \frac{x^2+4}{3x^2-3}$

vertical asymptotes: $3x^2-3=0$

$3(x^2-1)=0$

$3(x-1)(x+1)=0$

$\boxed{x=1}\ \boxed{x=-1}$ vertical asymptotes horizontal asymptotes: $y = b$, where $b = \lim_{x\to\infty} f(x)$ or $b = \lim_{x\to-\infty} f(x)$

Find $\lim_{x\to\infty} \frac{\boxed{1}x^2+4}{\boxed{3}x^2-3} = \boxed{\frac{1}{3}}$ same

$\boxed{y = \frac{1}{3}}$ - horizontal asymptote.

#15. $f(x) = \begin{cases} x^2+1, & x<2 \\ x+2, & x \geq 2 \end{cases}$

- x^2+1 is continuous for all x, so $f(x)$ is continuous if $x<2$
- $x+2$ is continuous for all x, so $f(x)$ is continuous if $x>2$
- $x = 2$.

$\lim_{x\to2^-} f(x) = \lim_{x\to2^-} (x^2+1) = 5$

$\neq$

$\lim_{x\to2^+} f(x) = \lim_{x\to2^+} (x+2) = 4$

$\boxed{f(x) \text{ is discontinuous at } x=2}$

#16. $s(t) = 1 + 2t + \frac{t^2}{4}$

(a) $V_{ave} = \frac{s(2)-s(1)}{2-1} = \frac{1+2(2)+\frac{2^2}{4} - (1+2+\frac{1}{4})}{1} = \boxed{\frac{11}{4}}$

(b) $v = s'(t) = 2 + \frac{2t}{4} = 2 + \frac{t}{2}$

$v(1) = 2 + \frac{1}{2} = \boxed{\frac{5}{2}}$

#17. $y = 2x^2-3$, $(\underbrace{2}_{x_0}, \overbrace{5}^{y_0})$

equation: $y - y_0 = m(x - x_0)$

$m = f'(2)$, where $f(x) = 2x^2-3$

$f'(x) = 4x$, $f'(2) = \boxed{8 = m}$

$y - 5 = 8(x-2)$

$\boxed{y = 8x - 11}$