

1. Let

$$p = 25 - 0.01x \quad \text{and} \quad C(x) = 2x + 9000$$

where  $0 \leq x \leq 2500$ , be the price-demand equation and cost function, respectively, for the manufacture of umbrellas.

- (a) Find the exact cost of producing the 31st umbrella.
  - (b) Use the marginal cost to approximate the cost of producing the 31st umbrella.
2. Find the equation of the tangent line to the graph of the function  $f(x) = \ln(1 - x^2 + 2x^4)$  at the point where  $x = 1$ .
3. Find the value(s) of  $x$  where the tangent line to the graph of the function  $y = 5e^{x^2 - 4x + 1}$  is horizontal.
4. Find each derivative

(a)  $\frac{d}{dx} \log_3(\sqrt[4]{4x^3 + 5x + 7})$

(b)  $\frac{d}{dx} 8^{1-2x^3}$

(c)  $\frac{d}{dx} \frac{3x^2}{(x^2 + 5)^3}$

(d)  $\frac{d}{dx} [(x^2 + x - 3)e^{2x+3}]$

5. Given the price-demand equation

$$0.02x + p = 60$$

- (a) Find the elasticity of demand  $E(p)$ .
  - (b) For which values of  $p$  is the revenue maximized? Find the maximum revenue.
  - (c) If  $p = \$10$  and the price is increased by 5%, what is the approximate change in demand?
  - (d) If  $p = \$40$  and the price is decreased, will revenue increase or decrease?
6. Find  $f''(x)$  for the functions
- (a)  $f(x) = x^2(2x^3 - 5)^4$
  - (b)  $f(x) = \frac{2}{x} - \frac{6}{x^3}$
7. Find the limit

(a)  $\lim_{x \rightarrow \infty} (x - x^3)$

(b)  $\lim_{x \rightarrow -\infty} (x - x^3)$

- (c)  $\lim_{x \rightarrow -\infty} \frac{x^3 - 1}{2x^4 + 1}$
- (d)  $\lim_{x \rightarrow \infty} \frac{x^4 - 1}{1 - x^3}$
- (e)  $\lim_{x \rightarrow \infty} \frac{x^4 - 2x^2 + 3}{1 - x^4}$
- (f)  $\lim_{x \rightarrow -\infty} \frac{e^{-2x} + 4}{3 - 2e^{-2x}}$

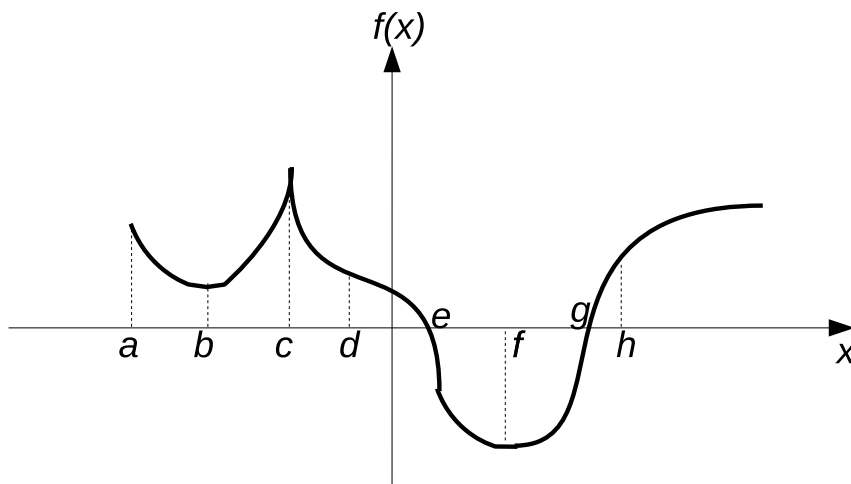
8. Find the horizontal asymptotes of the function

(a)  $f(x) = \frac{x^4 + 3x}{2 - 6x^4}$

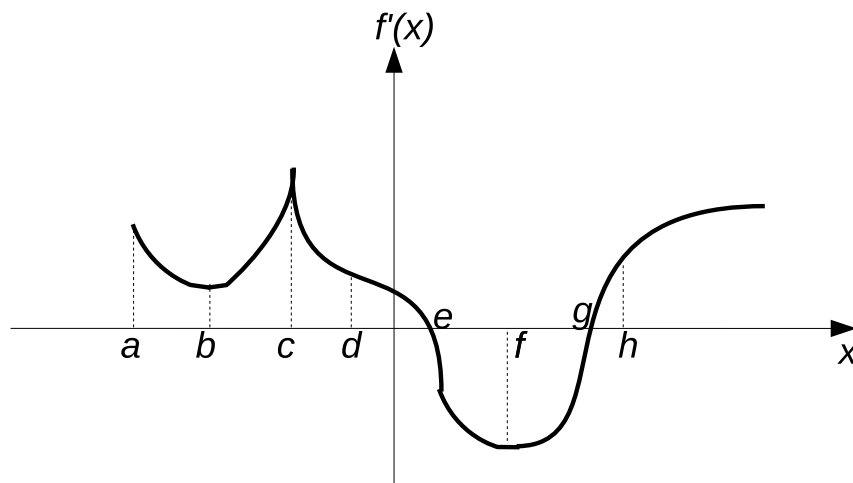
(b)  $f(x) = \frac{x}{x^2 + 4}$

9. Find the local extrema and the inflection points for the function  $f(x) = ax^3 + bx^2 + cx + d$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  are arbitrary constants.

10. Given the graph of the function  $y = f(x)$ .



- (a) Find the intervals on which  $f'(x) > 0$ .
- (b) Find the intervals on which  $f'(x) < 0$ .
- (c) Find  $x$ -coordinates of the points where  $f'(x) = 0$ .
- (d) Find the intervals on which  $f''(x) > 0$ .
- (e) Find the intervals on which  $f''(x) < 0$ .
- (f) Find  $x$ -coordinates of the points where  $f''(x) = 0$ .
11. Given the graph of the derivative  $f'(x)$  of the function  $f(x)$ .



- (a) Find the intervals on which  $f(x)$  is increasing/decreasing.
- (b) Find  $x$ -coordinates of the critical values for  $f(x)$ .
- (c) Find the intervals on which  $f(x)$  is concave up/concave down.
- (d) Find  $x$ -coordinates of the inflection points.
12. Given the function  $f(x) = \frac{1}{4}x^4 - 4x$ .
- (a) Find critical values of  $f(x)$ .
- (b) Find intervals on which  $f(x)$  is increasing and decreasing.
- (c) Find local extrema for  $f(x)$ .
- (d) Find intervals on which  $f(x)$  is concave upward and concave downward.
- (e) Find all inflection points of  $f(x)$ .
13. Find the absolute maximum and absolute minimum for the function  $f(x) = \sqrt{9 - x^2}$  on the interval  $[-1, 2]$ .
14. Find the absolute maximum and minimum for the function  $f(x) = \frac{x^2 - 1}{x^2 + 1}$ .
15. A box with a square base and open top must have a volume of  $32000 \text{ cm}^3$ . Find the dimensions of the box that minimize the amount of material used.