1. Let
$$p = 25 - 0.01x \qquad \text{and} \qquad C(x) = 2x + 9000$$
where $0 \le x \le 2500$, be the price-demand equation and cost function, respectively, for the manufacture of umbrellas.

   (a) Find the exact cost of producing the 31st umbrella.

   (b) Use the marginal cost to approximate the cost of producing the 31st umbrella.

2. Find the equation of the tangent line to the graph of the function $f(x) = \ln(1 - x^2 + 2x^4)$ at the point where $x = 1$.

3. Find the value(s) of x where the tangent line to the graph of the function $y = 5e^{x^2-4x+1}$ is horizontal.

4. Find each derivative

   (a) $\dfrac{d}{dx}\log_3(\sqrt[4]{4x^3 + 5x + 7})$

   (b) $\dfrac{d}{dx}8^{1-2x^3}$

   (c) $\dfrac{d}{dx}\dfrac{3x^2}{(x^2+5)^3}$

   (d) $\dfrac{d}{dx}[(x^2 + x - 3)e^{2x+3}]$

5. Given the price-demand equation
$$0.02x + p = 60$$

   (a) Find the elasticity of demand $E(p)$.

   (b) For which values of p is the revenue maximized? Find the maximum revenue.

   (c) If $p = \$10$ and the price is increased by 5%, what is the approximate change in demand?

   (d) If $p = \$40$ and the price is decreased, will revenue increase or decrease?

6. Find $f''(x)$ for the functions

   (a) $f(x) = x^2(2x^3 - 5)^4$

   (b) $f(x) = \dfrac{2}{x} - \dfrac{6}{x^3}$

7. Find the limit

   (a) $\lim\limits_{x\to\infty}(x - x^3)$

   (b) $\lim\limits_{x\to-\infty}(x - x^3)$

(c) $\lim_{x\to-\infty} \dfrac{x^3-1}{2x^4+1}$

(d) $\lim_{x\to\infty} \dfrac{x^4-1}{1-x^3}$

(e) $\lim_{x\to\infty} \dfrac{x^4-2x^2+3}{1-x^4}$

(f) $\lim_{x\to-\infty} \dfrac{e^{-2x}+4}{3-2e^{-2x}}$

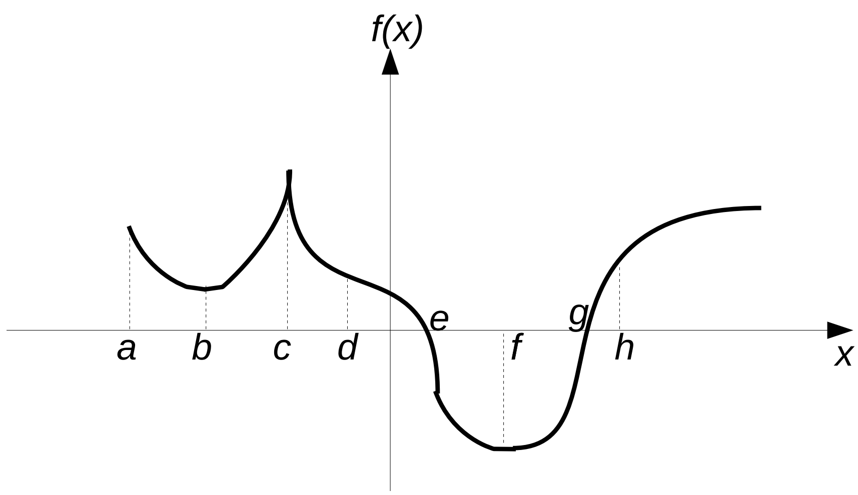
8. Find the horizontal asymptotes of the function

   (a) $f(x) = \dfrac{x^4+3x}{2-6x^4}$

   (b) $f(x) = \dfrac{x}{x^2+4}$

9. Find the local extrema and the inflection points for the function $f(x) = ax^3+bx^2+cx+d$, where a, b, c, and d are arbitrary constants.

10. Given the graph of the function $y = f(x)$.



   (a) Find the intervals on which $f'(x) > 0$.

   (b) Find the intervals on which $f'(x) < 0$.

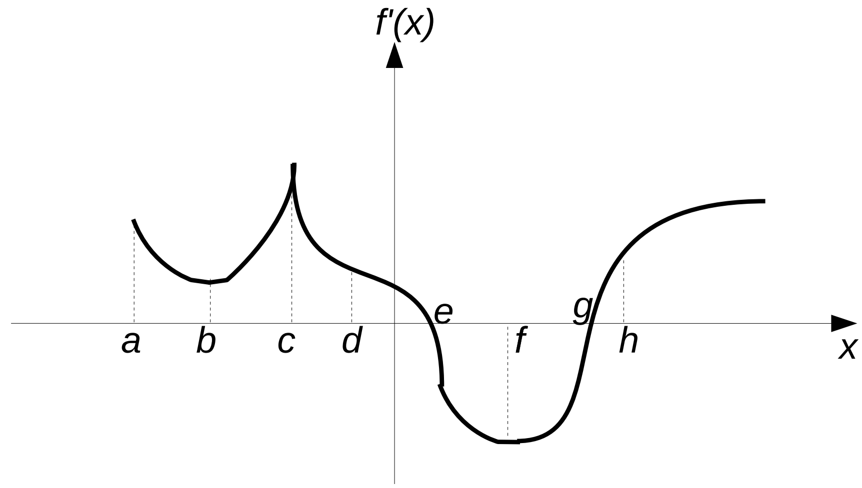
   (c) Find x-coordinates of the points where $f'(x) = 0$.

   (d) Find the intervals on which $f''(x) > 0$.

   (e) Find the intervals on which $f''(x) < 0$.

   (f) Find x-coordinates of the points where $f''(x) = 0$.

11. Given the graph of the derivative $f'(x)$ of the fuunction $f(x)$.



(a) Find the intervals on which $f(x)$ is increasing/decreasing.

(b) Find x-coordinates of the critical values for $f(x)$.

(c) Find the intervals on which $f(x)$ is convcave up/concave down.

(d) Find x-coordinates of the inflection points.

12. Given the function $f(x) = \dfrac{1}{4}x^4 - 4x$.

(a) Find critical values of $f(x)$.

(b) Find intervals on which $f(x)$ is increasing and decreasing.

(c) Find local extrema for $f(x)$.

(d) Find intervals on which $f(x)$ is concave upward and concave downward.

(e) Find all inflection points of $f(x)$.

13. Find the absolute maximum and absolute minimum for the function $f(x) = \sqrt{9 - x^2}$ on the interval $[-1, 2]$.

14. Find the absolute maximum and minimum for the function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

15. A box with a square base and open top must have a volume of 32000 cm^3. Find the dimensions of the box that minimize the amount of material used.

#1. a) Exact cost $= C(31) - C(30) = 2(31) + 9000 - 2(30) - 9000 = \boxed{\$2}$

b) $C'(x) = 2$

$C'(30) = \boxed{\$2}$

#2. $f(x) = \ln(1 - x^2 + 2x^4)$, $x = 1$

Tangent line: $y - f(1) = f'(1)(x - 1)$

$f(1) = \ln 2$

$f'(x) = \dfrac{1}{1 - x^2 + 2x^4}(1 - x^2 + 2x^4)' = \dfrac{-2x + 8x^3}{1 - x^2 + 2x^4}$; $f'(1) = \dfrac{6}{2} = 3$

Equation: $\boxed{y - \ln 2 = 3(x-1)}$ or $\boxed{y = 3x - 3 + \ln 2}$

#3. $y = 5e^{x^2-4x+1}$

$y' = 5e^{x^2-4x+1}(x^2-4x+1)' = 5(2x-4)e^{x^2-4x+1} = 0$

$2x - 4 = 0$ or $e^{x^2-4x+1} = 0$

$\boxed{x = 2}$ $\quad e^{x^2-4x+1}$ is never zero

#4. (a) $y = \log_3(\sqrt[4]{4x^3+5x+7}) = \log_3(4x^3+5x+7)^{1/4} = \frac{1}{4}\log_3(4x^3+5x+7)$

$y' = \frac{1}{4}\,\frac{(4x^3+5x+7)'}{(4x^3+5x+7)\ln 3} = \boxed{\frac{1}{4}\,\frac{12x^2+5}{(4x^3+5x+7)\ln 3}}$

(b) $(8^{1-2x^3})' = 8^{1-2x^3}(1-2x^3)'\ln 8 = \boxed{(-6x^2)\,8^{1-2x^3}\ln 8}$

(c) $\frac{d}{dx}\left[\frac{3x^2}{(x^2+5)^3}\right] = \frac{(3x^2)'(x^2+5)^3 - 3x^2((x^2+5)^3)'}{[(x^2+5)^3]^2}$

$= \frac{6x(x^2+5)^3 - 3x^2\,3(x^2+5)^2(x^2+5)'}{(x^2+5)^6} = \frac{6x(x^2+5)^3 - 9x^2(x^2+5)^2(2x)}{(x^2+5)^6}$

$= \frac{\cancel{(x^2+5)^2}[6(x)(x^2+5) - 18x^3]}{(x^2+5)^{\cancel{6}\,4}} = \frac{6x^3+30x-18x^3}{(x^2+5)^4} = \boxed{\frac{30x-12x^3}{(x^2+5)^4}}$

(d) $\frac{d}{dx}\left[(x^2+x-3)e^{2x+3}\right] = (x^2+x-3)'e^{2x+3} + (x^2+x-3)(e^{2x+3})'$

$= (2x+1)e^{2x+3} + (x^2+x-3)e^{2x+3}(2x+3)' = (2x+1)e^{2x+3} + 2(x^2+x-3)e^{2x+3}$

$= (2x^2+2x-6+2x+1)e^{2x+3} = \boxed{(2x^2+4x-5)e^{2x+3}}$

#5. $0.02x + p = 60$

$0.02x = 60 - p$

$x = \frac{60}{0.02} - \frac{1}{0.02}p$

$x = 3000 - 50p = f(p)$

(a) $E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(3000-50p)'}{3000-50p} = \frac{-p(-50)}{3000-50p} = \frac{50p}{3000-50p}$

$= \frac{p}{60-p}$

(b) $E(p) = 1$, $\frac{p}{60-p} = 1$

$p = 60 - p$

$2p = 60$

$p = \boxed{\$30}$

max Revenue $= pf(p) = 30f(30) = 30(3000 - 50(30)) = \boxed{\$45000}$

(c) (change in demand) $= E(p)$(change in price)

$p = 10$, $E(10) = \frac{10}{60-10} = \frac{10}{50} = 0.2 < 1$

demand is inelastic

change in demand $= 0.2 \cdot 5\% = 1\%$

$\boxed{\text{the demand will increase by } 1\%}$

(d) $p = 40$

$E(40) = \frac{40}{60-40} = \frac{40}{20} = 2 > 1$

demand is elastic

$\boxed{\text{the revenue will increase}}$

#6. (a) $f(x) = x^2(2x^3-5)^4$

$f'(x) = 2x(2x^3-5)^4 + (x^2)(4)(2x^3-5)^3(2x^3-5)'$

$= 2x(2x^3-5)^4 + 4x^2(2x^3-5)^3(6x^2) = 2x(x^3-5)^4 + 24x^4(2x^3-5)^3$

$f''(x) = 2(2x^3-5)^4 + 2x(4)(2x^3-5)^3(2x^3-5)' + 96x^3(2x^3-5)^3 + 24x^4(3)(2x^3-5)^2(2x^3-3)'$

$= 2(2x^3-5)^4 + 8x(2x^3-5)^3(6x^2) + 96x^3(2x^3-5)^3 + 72x^4(2x^3-5)^2(6x^2)$

$= \boxed{2(2x^3-5)^4 + 48x^3(2x^3-5)^3 + 96x^3(2x^3-5)^3 + 432x^4(2x^3-5)^2}$

(b) $f(x)=\frac{2}{x}-\frac{6}{x^3}=2x^{-1}-6x^{-3}$

$f'(x)=2(-1)x^{-1-1}-6(-3)x^{-3-1}=-2x^{-2}+18x^{-4}$

$f''(x)=-2(-2)x^{-2-1}+18(-4)x^{-4-1}$

$=\boxed{4x^{-3}-72x^{-5}}$

#7. (a) $\lim\limits_{x\to\infty}(x\boxed{-1}x^3)=\boxed{-\infty}$

(b) $\lim\limits_{x\to-\infty}(x\boxed{-1}x^3)=\boxed{\infty}$

(c) $\lim\limits_{x\to\infty}\frac{\boxed{x^3}-1}{\boxed{2x^4}+1}=\lim\limits_{x\to\infty}\frac{\cancel{x^3}}{2x^{\cancel{4}}}=\lim\limits_{x\to\infty}\frac{1}{2x}=\boxed{0}$

(d) $\lim\limits_{x\to\infty}\frac{\boxed{x^4}-1}{1\boxed{-x^3}}=\lim\limits_{x\to\infty}\frac{x^{\cancel{4}}}{\cancel{-x^3}}=\lim\limits_{x\to\infty}(-x)=\boxed{-\infty}$

(e) $\lim\limits_{x\to\infty}\frac{\boxed{x^4}-2x^3+3}{1\boxed{-x^4}}=\lim\limits_{x\to\infty}\frac{\cancel{x^4}}{-\cancel{x^4}}=\boxed{-1}$

(f) $\lim\limits_{x\to\infty}\frac{e^{-2x}+4}{3-2e^{-2x}}$ $\lim\limits_{x\to\infty}e^{-2x}=0$ $\boxed{\frac{4}{3}}$

$\lim\limits_{x\to-\infty}\frac{e^{-2x}+4}{3-2e^{-2x}}$ $\lim\limits_{x\to-\infty}e^{-2x}=\infty$ $\lim\limits_{x\to-\infty}\frac{\cancel{e^{-2x}}\left(1+\frac{4}{e^{-2x}}\right)}{\cancel{e^{-2x}}\left(\frac{3}{e^{-2x}}-2\right)}=\lim\limits_{x\to-\infty}\frac{1+4e^{2x}}{3e^{2x}-2}$

$\lim\limits_{x\to-\infty}e^{2x}=0$ $\frac{1}{-2}=\boxed{-\frac{1}{2}}$

#8. (a) $f(x)=\frac{x^4+3x}{2-6x^4}$

$\lim\limits_{x\to\infty}\frac{\boxed{x^4}+3x}{2\boxed{-6x^4}}=\lim\limits_{x\to\infty}\frac{\cancel{x^4}}{-6\cancel{x^4}}=-\frac{1}{6}$ $\boxed{H.A.\ y=-\frac{1}{6}}$

(b) $f(x)=\frac{x}{x^2+4}$

$\lim\limits_{x\to\infty}\frac{x}{x^2+4}=\lim\limits_{x\to\infty}\frac{x}{x^2}=\lim\limits_{x\to\infty}\frac{1}{x}=0$ $\boxed{H.A.\ y=0}$

#9. $f(x)=ax^3+bx^2+cx+d$

$f'(x)=3ax^2+2bx+c=0$

$x_1=\frac{-2b+\sqrt{4b^2-12ac}}{6a}=\frac{-2b+\sqrt{4(b^2-3ac)}}{6a}$

$$= \frac{-2b+\sqrt{4}\sqrt{b^2-3ac}}{6a} = \frac{-2b+2\sqrt{b^2-3ac}}{6a} = \frac{-b+\sqrt{b^2-3ac}}{3a}$$

$$x_2 = \frac{-b-\sqrt{b^2-3ac}}{3a}$$

(1) if $b^2-3ac>0$, then

$f'(x)$: $+$ | x_1 local max | $-$ | x_2 local min | $+$ if $a>0$

$f'(x)$: $-$ | x_1 local min | $+$ | x_2 local max | $-$ if $a<0$

(2) if $b^2-3ac=0$, then $x_1 = x_2 = -\frac{b}{3a}$

$f'(x)$: $-$ | $-\frac{b}{3a}$ local min | $+$ if $a>0$

$f'(x)$: $+$ | $-\frac{b}{3a}$ local max | $-$ if $a<0$

(3) if $b^2-3ac<0$, then there are no local extrema

$$f''(x) = 6ax+2b = 0$$

$$x = -\frac{2b}{6a} = \boxed{-\frac{b}{3a} \text{ - inflection point}}$$

#10. (a) $f(x)$ increases on $\boxed{(b,c)\cup(f,\infty)}$

(b) $f(x)$ decreases on $\boxed{(a,b)\cup(c,f)}$

(c) $\boxed{x=b,\ x=f}$

(d) f is CU on $\boxed{(a,d)\cup(e,g)}$

(e) f is CD on $\boxed{(d,e)\cup(g,\infty)}$

(f) inflection points @ $\boxed{x=d}$, $\boxed{x=e}$ and $\boxed{x=g}$

#11. (a) $\boxed{(a,e)\cup(g,\infty)}$

(b) $\boxed{x=e}$ $\boxed{x=g}$

(c) $f(x)$ is CU on $\boxed{(b,c)\cup(f,\infty)}$

$f(x)$ is CD on $\boxed{(a,b)\cup(c,f)}$

(d) $\boxed{x=c}$, $\boxed{x=f}$

#12. $f(x)=\frac{1}{4}x^4-4x$

(a) $f'(x)=\frac{1}{4}(4x^3)-4=x^3-4=0$

$x^3=4 \Rightarrow \boxed{x=\sqrt[3]{4}\approx 1.59}$ C.V.

(b) sign chart for $f'(x)$:

$f'(x)$: $-$ | $\sqrt[3]{4}$ | $+$

local min

$f'(0)=-1<0$

$f'(2)=8-1=7>0$

$$\boxed{\begin{array}{l} f \text{ decreases on } (-\infty,\sqrt[3]{4}) \\ f \text{ increases on } (\sqrt[3]{4},\infty) \end{array}}$$

(c) f has the local min @ $x=\sqrt[3]{4}$

no local max

(d) $f''(x)=3x^2=0$

$x=0$

sign chart for $f''(x)$: $+$ ($\cup$) | 0 | $+$ ($\cup$)

$f''(1)=3>0$

$f''(-1)=3>0$

$f(x)$ is CU on $\boxed{(-\infty,0)\cup(0,\infty)}$

(e) no inflection points.

#13. $f(x)=\sqrt{9-x^2}=(9-x^2)^{1/2}$ on $[-1,2]$

$$f'(x)=\frac{1}{2}(9-x^2)^{-1/2}(9-x^2)'=\frac{1}{2}(9-x^2)^{-1/2}(-2x)$$

$$=-x(9-x^2)^{-1/2}=-\frac{x}{\sqrt{9-x^2}}=0$$

$x=0$

$9-x^2\neq 0,\quad x\neq 3,\quad x\neq -3$

$x=0$, $\boxed{x=3,\ x=-3 \text{ not in } [-1,2]}$ are critical values

$f(0)=\sqrt{9-0}=\boxed{3 \text{ abs. max}}$

$f(-1)=\sqrt{9-(-1)^2}=\sqrt{9-1}=\sqrt{8}\approx 2.83$

$f(2)=\sqrt{9-2^2}=\sqrt{9-4}=\boxed{\sqrt{5}\approx 2.24 \text{ abs. min}}$

#14. $f(x)=\dfrac{x^2-1}{x^2+1}$

$$f'(x)=\frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2}=0$$

$x=0$

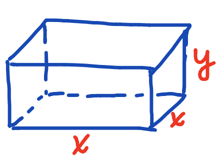
$$f''(x)=\frac{4(x^2+1)^2-(4x)2(x^2+1)(x^2+1)'}{[(x^2+1)^2]^2}=\frac{4(x^2+1)^2-8x(x^2+1)(2x)}{(x^2+1)^4}=\frac{\cancel{(x^2+1)}[4(x^2+1)-16x^2]}{(x^2+1)^{\cancel{4}\,3}}$$

$$=\frac{4x^2+4-16x^2}{(x^2+1)^3}=\frac{4-12x^2}{(x^2+1)^3}$$

$$f''(0)=\frac{4}{1}=4>0$$

$\boxed{\text{f has the absolute min @ } x=0 \text{ no absolute max}}$ $f(0)=\boxed{-1 \text{ - abs. min value}}$

#15.



the side of the base is of length x

the height is y

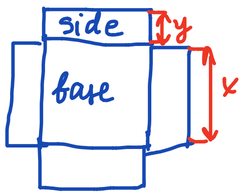
$V=x^2y=32{,}000$

$$y=\frac{32{,}000}{x^2}$$

amount of material used

$$S = 4(xy) + x^2 = 4x \frac{32,000}{x^2} + x^2$$

$$= \frac{128,000}{x} + x^2$$ - to be minimized



$$S'(x) = -\frac{128,000}{x^2} + 2x = \frac{2x^3 - 128,000}{x^2} = 0$$

$$2x^3 - 128,000 = 0$$

$$x^3 = \frac{128,000}{2} = 64000$$

$$x = \sqrt[3]{64000} = 40$$

$$y = \frac{32,000}{x^2} = \frac{32,000}{(40)^2} = \frac{32000}{1600} = 20$$

$$\boxed{40 \times 40 \times 20}$$