Sample problems for Test 3

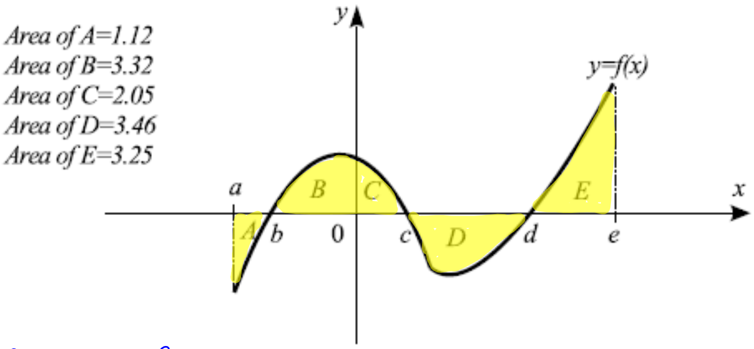
1. Let $\int_1^5 f(x)dx = 2$, $\int_1^3 g(x)dx = -1$, and $\int_3^5 g(x)dx = 4$. Find $\int_5^1 (4g(x) - 3f(x))dx$.

$$\int_5^1 (4g(x) - 3f(x))dx = -\int_1^5 (4g(x) - 3f(x))\,dx$$

$$= -\Big(4\underbrace{\int_1^5 g(x)dx}_{\underbrace{\int_1^3 g(x)dx + \int_3^5 g(x)dx}_{4+(-1)=3}} - 3\underbrace{\int_1^5 f(x)dx}_{2}\Big)$$

$$= -\big(4(3) - 3(2)\big) = \boxed{-6}$$

2. Calculate $2\int_a^c f(x)\,dx - \int_0^e f(x)\,dx$ by refering to the figure with the indicated areas.



$$2\int_a^c f(x)\,dx - \int_0^e f(x)\,dx = 2\left(-A(A)+A(B)+A(C)\right) - \left(A(C) - A(D) + A(E)\right)$$

$$= \ldots = 6.66$$

3. Find the following indefinite integrals:

(a) $2\int \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\, dx$ $\left| \begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2} x^{-1/2} dx \end{array} \right| = 2 \int e^u\, du$

$= 2e^u + C$

$= \boxed{2e^{\sqrt{x}} + C}$

(b) $\frac{1}{4}\int \frac{4(x^3 - 3)}{x^4 - 12x + 3}\,dx$

$u = x^4 - 12x + 3$

$du = (4x^3 - 12)\,dx$

$= 4(x^3 - 3)\,dx$

$= \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + C$

$= \boxed{\frac{1}{4}\ln|x^4 - 12x + 3| + C}$

(c) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$ $\quad \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array} \right| = \int u^2\,du = \frac{u^3}{3} + C$

$$= \boxed{\frac{(\ln x)^3}{3} + C}$$

(d) $-\frac{1}{4}\int \frac{-4x}{(5-2x^2)^5}\,dx$ $\quad \left| \begin{array}{l} u = 5-2x^2 \\ du = -4x\,dx \end{array} \right|$

$$= -\frac{1}{4}\int \frac{du}{u^5} = -\frac{1}{4}\int u^{-5}\,du = -\frac{1}{4}\,\frac{u^{-5+1}}{-5+1} + C$$

$$= -\frac{1}{4}\,\frac{u^{-4}}{-4} + C$$

$$= \boxed{\frac{1}{16}(5-2x^2)^{-4} + C}$$

(e) $\int \frac{x}{\sqrt{3-x}}\,dx$ $\quad\Big|\quad u = 3-x \Rightarrow x = 3-u$; $\quad du = -dx \Rightarrow dx = -du \quad\Big|$

$$= \int \frac{3-u}{\sqrt{u}}(-du) = -\int (3-u)u^{-1/2}\,du$$

$$= -\int (3u^{-1/2} - u^{1/2})\,du$$

$$= -\left(3\,\frac{u^{-1/2+1}}{-1/2+1} - \frac{u^{1/2+1}}{1/2+1}\right) + C$$

$$= -\left(3(2)u^{1/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \boxed{-6(3-x)^{1/2} - \frac{2}{3}(3-x)^{3/2} + C}$$

(f) $\frac{1}{2}\int 2e^{2x-1}\,dx$

$$\left[\begin{array}{l} u = 2x-1 \\ du = 2\,dx \end{array}\right.$$

$$= \frac{1}{2}\int e^{u}\,du = \frac{1}{2}e^{u} + C$$

$$= \boxed{\frac{1}{2}e^{2x-1} + C}$$

4. The weekly marginal revenue from the sale of x pairs of tennis shoes is given by

$$R'(x) = 40 - 0.002x + \frac{200}{x+1}, \quad R(0) = 0,$$

where $R(x)$ is revenue in dollars. Find the revenue function $R(x)$. Find the revenue from the sale of 1000 pairs of shoes.

$$R(x) = \int R'(x)dx = \int \left(40 - 0.002x + \frac{200}{x+1}\right) dx$$

$$= 40\int dx - 0.002\int x dx + 200\int \frac{dx}{x+1} \qquad \left| \begin{array}{l} u = x+1 \\ du = dx \end{array} \right|$$

$$= 40x - 0.002\frac{x^2}{2} + 200\int \frac{du}{u}$$

$$= 40x - 0.001x^2 + 200 \ln|u| + C$$

$$= 40x - 0.001x^2 + 200 \ln|x+1| + C$$

$$R(0) = 0$$

$$R(0) = 0 - 0 + 200 \overset{0}{\ln 1} + C$$

$$= \boxed{C = 0}$$

$$\boxed{R(x) = 40x - 0.001x^2 + 200 \ln|x+1|}$$

$$R(1000) = 40(1000) - 0.001(1000)^2 + 200\ln(1001) = \boxed{\$ 40381.75}$$

5. Use L_6 and R_6 to approximate $\int_2^5 (0.25x^2 - 4)dx$.

Approximate the distance traveled by an object from $t=2$ sec to $t=5$ sec, if its velocity is given by $v(t) = 0.025t^2 - 4$. Brake $[2,5]$ into 6 subintervals.

Use L_6 and R_6 to approximate the area of the region that lies below the graph of $f(x) = 0.25x^2 - 4$ and above the x-axis between $x=2$ and $x=5$.

Step 1. Find change is $x = \Delta x = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$

Step 2. Find the partition points:

$x_0 = 2$	$f(2) = -3$
$x_1 = 2 + 0.5 = 2.5$	$f(2.5) = -2.438$
$x_2 = 2.5 + 0.5 = 3$	$f(3) = -1.75$
$x_3 = 3 + 0.5 = 3.5$	$f(3.5) = -.9375$
$x_4 = 3.5 + 0.5 = 4$	$f(4) = 0$
$x_5 = 4 + 0.5 = 4.5$	$f(4.5) = 1.0625$
$x_6 = 5$	$f(5) = 2.25$

$L_6 = [f(2) + f(2.5) + f(3) + f(3.5) + f(4) + f(4.5)]\overbrace{\Delta x}^{1/2}$

$R_6 = [f(2.5) + f(3) + f(3.5) + f(4) + f(4.5) + f(5)]\underbrace{\Delta x}_{1/2}$

6. Evaluate each of the definite integrals:

(a) $\int_0^A 32(x^2+1)^7 x\,dx \quad (A>0)$

$$\left| \begin{array}{l} u = x^2+1 \\ du = 2x\,dx \end{array} \right|$$

$$= \int_{x=0}^{x=A} 16\cdot 2\,(x^2+1)^7\,x\,dx = 16\int_{x=0}^{x=A} u^7\,du = 16\,\frac{u^8}{8}\Big]_{x=0}^{x=A}$$

$$= 2\,(x^2+1)^8\Big]_0^A = \boxed{2\left((A^2+1)^8-1\right)}$$

(b) $\int_B^2 \left(5x - 4\frac{x^2}{\sqrt[4]{x^3}}\right) dx, \quad (0 < B < 2)$

$$= \int_B^2 \left(5x - 4x^{2-3/4}\right) dx = \int_B^2 \left(5x - 4x^{5/4}\right) dx$$

$$= 5\int_B^2 x\, dx - 4\int_B^2 x^{5/4}\, dx$$

$$= 5\,\frac{x^2}{2}\Big]_B^2 - 4\,\frac{x^{5/4+1}}{5/4+1}\Big]_B^2$$

$$= \frac{5}{2}\left(4 - B^2\right) - 4\cdot\frac{4}{9}\,x^{9/4}\Big]_B^2 = \boxed{\frac{5}{2}\left(4 - B^2\right) - \frac{16}{9}\left(2^{9/4} - B^{9/4}\right)}$$

B

7. Find the average value of the function $f(x) = 4x - 3x^2$ over the interval $[-2, 2]$.

$$f_{ave} = \frac{1}{2-(-2)} \int_{-2}^{2} (4x - 3x^2)\,dx$$

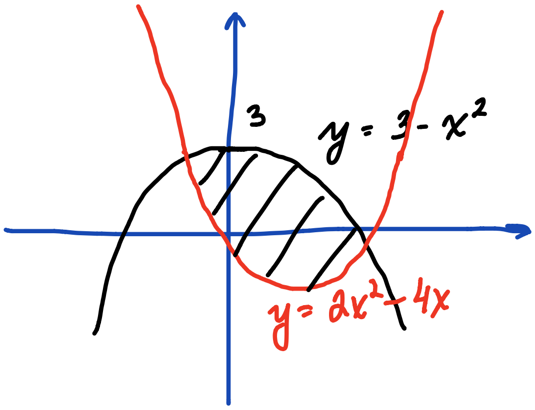
$$= \frac{1}{4} \int_{-2}^{2} (4x - 3x^2)\,dx = \frac{1}{4}\left(4\frac{x^2}{2} - \frac{3x^3}{3}\right)_{-2}^{2}$$

$$= \frac{1}{4}\left(2x^2 - x^3\right)_{-2}^{2} = \frac{1}{4}\left(2\left(2^2 - (-2)^2\right) - \left(2^3 - (-2)^3\right)\right)$$

$$= -\frac{1}{4}(16) = \boxed{-4}$$

8. Find the area of the region bounded by:

(a) $y = 3 - x^2$, $y = 2x^2 - 4x$



Intersection points:

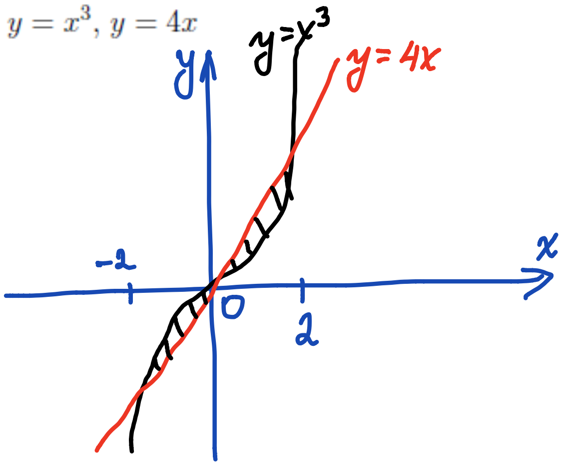
$(-0.535, 2.714)$

and $(1.869, 0.491)$

$$A = \int_{-0.535}^{1.869} \left[(3 - x^2) - (2x^2 - 4x)\right] dx$$

$$\approx \boxed{6.944}$$

(b) $y = x^3$, $y = 4x$



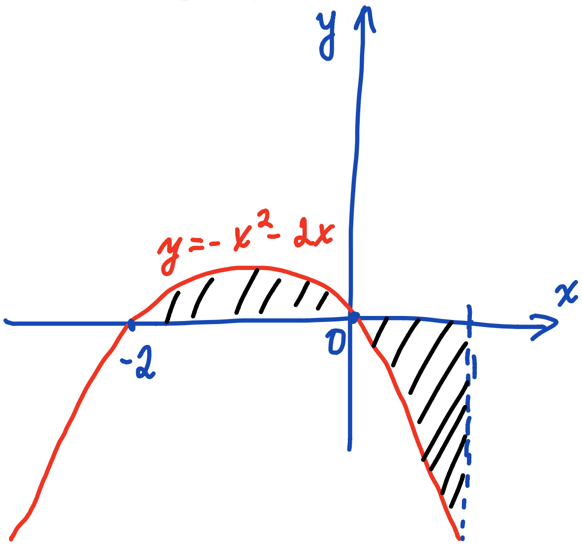
Intersection points:

$x_1 = -2,\ x_2 = 0,\ x_3 = 2$

$$A = \int_{-2}^{0} (x^3 - 4x)\,dx + \int_{0}^{2} (4x - x^3)\,dx$$

$$= \boxed{8}$$

(c) $y = -x^2 - 2x$, $y = 0$, $x = -2$, $x = 1$.



x-intercepts:

$-x^2 - 2x = 0$

$x(-x-2) = 0$

$x_1 = 0$ or $-x - 2 = 0$

$x_2 = -2$

$$A = \int_{-2}^{0} (-x^2 - 2x)\,dx + \int_{0}^{1} \left(0 - (-x^2 - 2x)\right) dx$$

$$= \boxed{\frac{8}{3}}$$

9. Find the consumers' surplus and producers' surplus at the equilibrium price level for the given price-demand and price-supply equations.

$$p = D(x) = 70 - 0.2x$$
$$p = S(x) = 13 + 0.0012x^2$$

Round all values to the nearest integer.

Equilibrium point:

$$70 - 0.2x = 13 + 0.0012x^2$$
$$0.0012x^2 + 0.2x - 57 = 0$$
$$x = \frac{-0.2 + \sqrt{0.04 + 4(57)(0.0012)}}{2(0.0012)} = 150$$

Equilibrium quantity $\bar{x} = 150$

Equilibrium price $\bar{p} = 70 - 0.2(150)$
$= 40$

consumers' surplus:

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]\,dx = \int_0^{150} [70 - 0.2x - 40]\,dx = \boxed{2250}$$

producers' surplus:

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]\,dx = \int_0^{150} [40 - (13 + 0.0012x^2)]\,dx = \boxed{2700}$$

10. Let $f(x,y) = 2x - 3y + 14$ and $g(x,y) = \dfrac{10}{x^2+4y}$. Find $f(2,-3) - 4g(-1,2)$.

$f(2,-3) = 2(2) - 3(-3) + 14 = 27$

$f(-1,2) = \dfrac{10}{(-1)^2 + 4(2)} = \dfrac{10}{9}$

$f(2,-3) - 4g(-1,2) = 27 - 4\,\dfrac{10}{9} = \dfrac{243-40}{9} = \boxed{\dfrac{203}{9}}$

11. Weston Publishing publishes a deluxe edition and a standard edition of its English language dictionary. Weston's management estimates that the number of deluxe editions demanded is x copies/day and the number of standard editions demanded is y copies/day when the unit prices are

$$p = 20 - 0.005x - 0.001y$$
$$q = 15 - 0.001x - 0.003y$$

dollars, respectively. Find the daily total revenue function $R(x, y)$. Evaluate $R(7, 3)$.

$$R(x,y) = xp + yq$$
$$= x(20 - 0.005x - 0.001y) + y(15 - 0.001x - 0.003y)$$
$$= 20x - 0.005x^2 - 0.001xy + 15y - 0.001xy - 0.003y^2$$
$$= 20x + 15y - 0.002xy - 0.005x^2 - 0.003y^2$$

$$R(7,3) = 20(7) + 15(3) - 0.002(7)(3) - 0.005(7^2) - 0.003(3^2)$$
$$= \boxed{184.686}$$

12. Find f_x and f_y for the functions:

(a) $f(x, y) = \dfrac{2xy}{1 + x^2y^2}$

$$f_x = \frac{2y(1+x^2y^2) - 2xy^2(2xy)}{(1+x^2y^2)^2} = \frac{2y + 2x^2y^3 - 4x^2y^3}{(1+x^2y^2)^2}$$

$$= \frac{2y - 2x^2y^3}{(1+x^2y^2)^2}$$

$$f_y = \frac{2x(1+x^2y^2) - 2x^2y(2xy)}{(1+x^2y^2)^2} = \frac{2x + 2x^3y^2 - 4x^3y^2}{(1+x^2y^2)^2}$$

$$= \frac{2x - 2x^3y^2}{(1+x^2y^2)^2}$$

(b) $f(x, y) = \sqrt{2x - y^2} = (2x - y^2)^{1/2}$

$f_x = \frac{1}{2}(2x - y^2)^{\frac{1}{2}-1}(2x - y^2)'_x = \frac{1}{\cancel{2}}(2x - y^2)^{-1/2}(\cancel{2})$

$= (2x - y^2)^{-1/2}$

$f_y = \frac{1}{2}(2x - y^2)^{\frac{1}{2}-1}(2x - y^2)'_y = \frac{1}{\cancel{2}}(2x - y^2)^{-1/2}(-\cancel{2}y)$

$= -y(2x - y^2)^{-1/2}$

(c) $f(x,y) = xe^{x\sqrt{y}}$

$$f_x = e^{x\sqrt{y}} + x e^{x\sqrt{y}} (x\sqrt{y})'_x = e^{x\sqrt{y}} + xe^{x\sqrt{y}} (\sqrt{y})$$

$$= e^{x\sqrt{y}} + x\sqrt{y}\, e^{x\sqrt{y}}$$

$$f_y = x e^{x\sqrt{y}} (x\sqrt{y})'_y = xe^{x\sqrt{y}}\, x \frac{1}{2} y^{-1/2}$$

$$= \frac{1}{2} x^2 y^{-1/2} e^{x\sqrt{y}}$$

13. Find f_x, f_y, f_{xx}, f_{xy}, and f_{yy} for the function $f(x,y) = -4x^3y^5 + 9x^6y^2$.

$f_x = -4(3)x^2y^5 + 9(6)x^5y^2 = -12x^2y^5 + 54x^5y^2$

$f_y = -4x^3(5)y^4 + 9x^6(2y) = -20x^3y^4 + 18x^6y$

$f_{xx} = (f_x)'_x = -12(2x)y^5 + 54(5)x^4y^2$

$= -24xy^5 + 270x^4y^2$

$f_{yy} = (f_y)'_y = -20x^3(4y^3) + 18x^6 = -80x^3y^3 + 18x^6$

$f_{xy} = (f_y)'_x = (f_x)'_y = -12x^2(5)y^4 + 54x^5(2y)$

$= -60x^2y^4 + 108x^5y$