MATH 142, Section ______ Quiz 4 (Sections 4.4, 4.5, 5.1)

Due Friday, Oct. 17, 2014 at the beginning of class

NAME (print): key

Instructions:

- For problems 1 – 3 mark only one choice. Wrong answer will receive no credit. Each problem is worth 2 points.
- For Problem 4 present your solutions in the space provided. **Show all your work** neatly and concisely and **clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading to it. Credit will not be given for an answer not supported by work.
- STAPLE ALL THE SHEETS.

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1. Find the derivative of the function $f(x) = \log_2 [x^6(5x^3+4)^9]$.

   (a) $\dfrac{975x^7(5x^3+4)^8}{x^6(5x^3+4)^9 \ln 2}$

   (b) $\dfrac{975x^7(5x^3+4)^8}{x^6(5x^3+4)^9}$

   (c) $\dfrac{165x^3+24}{x(5x^3+4)}$

   (d) $\dfrac{165x^3+24}{x(5x^3+4)\ln 2}$

   (e) None of these

$$= \log_2(x^6) + \log_2(5x^3+4)^9$$
$$= 6\log_2 x + 9\log_2(5x^3+4)$$
$$f'(x) = \frac{6}{x\ln 2} + \frac{9(5x^3+4)'}{(5x^3+4)\ln 2}$$
$$= \frac{6}{x\ln 2} + \frac{9(15x^2)}{(5x^3+4)\ln 2}$$
$$= \frac{6(5x^3+4) + 135x^3}{x(5x^3+4)\ln 2} = \boxed{\frac{165x^3+24}{x(5x^3+4)\ln 2}}$$

2. Find the derivative of the function $f(x) = 3^{(x^2+2x)(x-3)^3}$.

   (a) $[(5x^2+2x-6)(x-3)^2]\, 3^{(x^2+2x)(x-3)^3} \ln 3$

   (b) $3^{(x^2+2x)(x-3)^3} \ln 3$

   (c) $[6(x+1)(x-3)^2]\, 3^{(x^2+2x)(x-3)^3} \ln 3$

   (d) $[6(x+1)(x-3)^2]\, 3^{(x^2+2x)(x-3)^3}$

   (e) None of these

$$f'(x) = 3^{(x^2+2x)(x-3)^3}\left[(x^2+2x)(x-3)^3\right]' \ln 3$$
$$= 3^{(x^2+2x)(x-3)^3}\left[(2x+2)(x-3)^3 + (x^2+2x)3(x-3)^2\right] \ln 3$$
$$= 3^{(x^2+2x)(x-3)^3} \ln 3\, (x-3)^2 \left[(2x+2)(x-3) + 3(x^2+2x)\right]$$
$$= 3^{(x^2+2x)(x-3)^3} \ln 3\, (x-3)^2 \left[2x^2-6x+2x-6 + 3x^2+6x\right]$$
$$= \boxed{3^{(x^2+2x)(x-3)^3} \ln 3\, (x-3)^2\, (5x^2+2x-6)}$$

[more problems on back]

3. Given the price-demand equation

$$p = 40 - 0.2x$$

Find the elasticity of demand at the price level $p = 10$.

(a) $\frac{1}{19}$

(b) 40

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(e) None of these

$p = 40 - 0.2x$

$p - 40 = -0.2x$

$x = -\frac{1}{0.2}p - \frac{40}{-0.2}$

$= -5p + 200 = f(p)$

$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(-5)}{-5p+200} = \frac{5p}{200-5p} = \frac{p}{40-p}$

$E(10) = \frac{10}{40-10} = \boxed{\frac{1}{3}}$

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4. *[4 pts]* Find the critical values of f, the intervals on which f is increasing, the intervals on which f is decreasing, and the local extrema for the function $f(x) = 2x^3 - 3x^2 - 36x$. **Do not graph.**

[0.5pt]

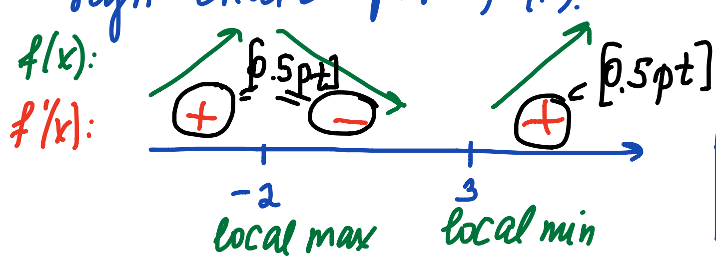
$f'(x) = 6x^2 - 6x - 36 = 0$

$6(x^2 - x - 6) = 0$

$6(x-3)(x+2) = 0$

critical values: $x_1 = -2$, $x_2 = 3$ [0.5pt] [0.5pt]

sign chart for $f'(x)$:



$f'(-3) = 6(-3)^2 - 6(-3) - 36 = 36 > 0$

$f'(0) = -36 < 0$

$f'(4) = 6(4)^2 - 6(4) - 36 = 36 > 0$

f increases on $(-\infty, -2) \cup (3, \infty)$ [0.25pt]

f decreases on $(-2, 3)$ [0.25pt]

f has the local max @ $x = -2$ [0.25pt]

f has the local min @ $x = 3$ [0.25pt]