MATH 142, Section _____ Quiz 5 (Sections 5.2, 5.3, 5.4)

Due Friday, Oct. 24, 2014 at the beginning of class

NAME (print): key

Instructions:

- For problems 1 – 2 mark only one choice. Wrong answer will receive no credit. Each problem is worth 2 points.
- For Problem 4 present your solutions in the space provided. **Show all your work** neatly and concisely and **clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading to it. Credit will not be given for an answer not supported by work.
- STAPLE ALL THE SHEETS.

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1. Find the limit $\lim_{x\to\infty} \dfrac{1+e^{3x}}{1-2e^{3x}}$.

$\lim_{x\to\infty} e^{3x} = \infty$

$= \lim_{x\to\infty} \dfrac{e^{3x}\left(\frac{1}{e^{3x}}+1\right)}{e^{3x}\left(\frac{1}{e^{3x}}-2\right)} = \lim_{x\to\infty} \dfrac{e^{-3x}+1}{e^{-3x}-2}$

$\lim_{x\to\infty} e^{-3x} = 0$, thus

$= -\frac{1}{2}$

(a) 1

(b) $-\frac{1}{2}$

(c) 0

(d) $-\infty$

(e) None of these

2. Find all horizontal asymptots for $f(x) = \dfrac{3x^3+3x-5}{2-x^3}$

$\lim_{x\to\infty} \dfrac{3x^3+3x-5}{2-x^3} = \lim_{x\to\infty} \dfrac{3x^3}{-x^3} = -3$

H.A. $y = -3$

(a) $y = 0$

(b) $y = 3$

(c) $y = -3$

(d) $y = -\frac{5}{2}$

(e) There are no horizontal asymptotes

[more problems on back]

3. *[6 pts]* Use calculus, to sketch $y = \dfrac{x}{(x-1)^2}$. $= x(x-1)^{-2}$

[0.25] Domain: $x \neq 1$; Range: $(-\infty, \infty)$; Intercepts: $(0,0)$ [0.25]

V.A. $x=1$ [0.25]

H.A. $\lim\limits_{x\to\infty} \dfrac{x}{(x-1)^2} = \lim\limits_{x\to\infty} \dfrac{x}{x^2} = 0$

H.A. $y=0$ [0.25]

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2. $f'(x) = (x)'(x-1)^{-2} + x((x-1)^{-2})' = (x-1)^{-2} + x(-2)(x-1)^{-3}$

$= \dfrac{1}{(x-1)^2} + \dfrac{-2x}{(x-1)^3} = \dfrac{x-1-2x}{(x-1)^3} = \boxed{\dfrac{-x-1}{(x-1)^3}}$ [1pt] $= 0$

$-x-1=0 \Rightarrow \boxed{x=-1}$ [0.25] - critical value

$x \neq 1$

sign chart for f':

f: ↗ ↗ ↘

f': $-$ $+$ $-$

-1 | 1

-1 local min

$f(-1) = \dfrac{-1}{(-1-1)^2} = -\dfrac{1}{4}$

f has the local min @ $(-1, -1/4)$; no local max [0.25]

$f'(0) = -\dfrac{1}{(-1)^3} = 1 > 0$

$f'(2) = \dfrac{-3}{1^3} = -3 < 0$

$f'(-2) = \dfrac{2-1}{(-3)^3} = \dfrac{1}{-27} < 0$

f increases on $(-1, 1)$ [0.25]

f decreases on $(-\infty, -1) \cup (1, \infty)$ [0.25]

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3. $f''(x) = [(-x-1)(x-1)^{-3}]' = (-x-1)'(x-1)^{-3} + (-x-1)[(x-1)^{-3}]'$

$= -(x-1)^{-3} + (-x-1)(-3)(x-1)^{-4} = -\dfrac{1}{(x-1)^3} + \dfrac{3x+3}{(x-1)^4}$

$= \dfrac{-(x-1)+3x+3}{(x-1)^4} = \boxed{\dfrac{2x+4}{(x-1)^4}}$ [1pt] $= 0$

$2x+4=0$, $\boxed{x=-2}$ [0.25]

$x \neq 1$

sign chart for f'':

f: ∩ ∪ ∪

f'': $-$ $+$ $+$

-2 | 1

-2 inflection point

$f(-2) = \dfrac{-2}{(-2-1)^2} = -\dfrac{2}{9}$

$f''(0) = \dfrac{4}{(-1)^4} = 4 > 0$

$f''(2) = \dfrac{8}{14} = 8 > 0$

$f''(-3) = \dfrac{-6+4}{(-4)^4} = \dfrac{-2}{196} < 0$

f is CD on $(-\infty, -2)$ [0.25]

f is CU on $(-2, 1) \cup (1, \infty)$ [0.25]

f has the inflection point @ $(-2, -2/9)$ [0.25]

[1pt] for the graph