MATH 142, Section ______ Quiz 6 (Sections 6.1 – 6.6)

Due Monday, Nov. 24, 2014 at the beginning of class

NAME (print): key

Instructions:

- For problems 1 – 7 mark only one choice. Wrong answer will receive no credit. Each problem is worth 2 points.
- For Problem 8 present your solutions in the space provided. **Show all your work** neatly and concisely and **clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading to it. Credit will not be given for an answer not supported by work.
- STAPLE ALL THE SHEETS.

---

1. Evaluate the integral $\int_1^{16} (e^x - \sqrt[4]{x^3})\, dx$

$$= \int_1^{16} e^x\,dx - \int_1^{16} x^{3/4}\,dx$$

$$= e^x\Big]_1^{16} - \frac{x^{3/4+1}}{3/4+1}\Big]_1^{16}$$

$$= e^{16} - e^1 - \frac{4}{7}x^{7/4}\Big]_1^{16}$$

$$= e^{16} - e - \frac{4}{7}\left(16^{7/4} - 1\right)$$

$$= e^{16} - e - \frac{4}{7}(128 - 1) = \boxed{e^{16} - e - \frac{508}{7}}$$

   (a) $e^{16} - e - \frac{508}{7}$
   (b) $e - e^{16} - \frac{516}{7}$
   (c) $e^{16} - e - \frac{3}{4}(16^{4/3} - 1)$
   (d) $e - e^{16} - \frac{3}{4}(16^{4/3} + 1)$
   (e) None of these

2. Find the average value of the function $f(x) = \frac{x}{\sqrt{x^2+1}}$ on the interval $[-2, 3]$. Round your answer to three decimal places.

   (a) 0.926
   (b) 0.185
   (c) 1.567
   (d) 1.952
   (e) None of these

$$f_{ave} = \frac{1}{3-(-2)} \int_{-2}^{3} \frac{x}{\sqrt{x^2+1}}\,dx$$

$$= \frac{1}{5}\int_{-2}^{3} \frac{x}{\sqrt{x^2+1}}\,dx \approx \boxed{\cancel{0.926}}$$

≈ 0.185

3. Approximate the area below the curve $y = 2x + 1$ between 1 and 3 by partitioning the interval in 4 pieces and using the right-hand sum R_4.

(a) 9

(b) 12.5

(c) 7.5

(d) 11

(e) None of these

$\Delta x = \frac{3-1}{4} = 0.5$

$x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, $x_4 = 3$

$f(1) = 3$, $f(1.5) = 4$, $f(2) = 5$, $f(2.5) = 6$, $f(3) = 7$

$R_4 = (f(1.5) + f(2) + f(2.5) + f(3))\Delta x$

$= (4+5+6+7)0.5$

$= \boxed{11}$

4. Let $\int_1^5 f(x)dx = 2$, $\int_1^3 g(x)dx = -1$, and $\int_3^5 g(x)dx = 4$. Find $\int_5^1 (4g(x) - 3f(x))dx$.

(a) 6

(b) -6

(c) 16

(d) 0

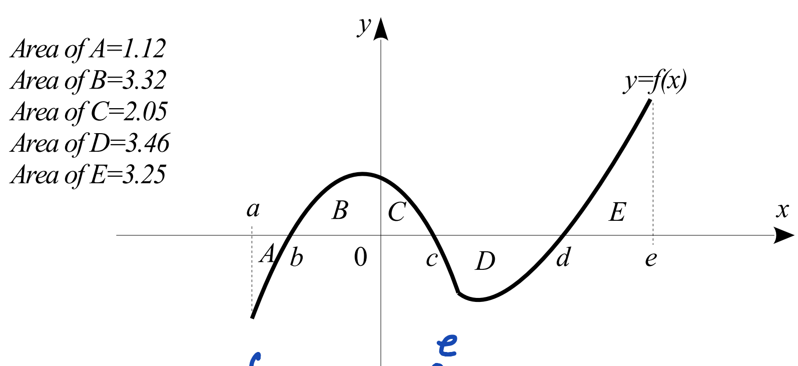
(e) None of these

$\int_5^1 [4g(x) - 3f(x)]dx = -4\int_1^5 g(x)dx + 3\int_1^5 f(x)dx$

$= -4\left[\int_1^3 g(x)dx + \int_3^5 g(x)dx\right] + 3\int_1^5 f(x)dx$

$= -4(-1+4) + 3(2) = \boxed{-6}$

5. Calculate $2\int_a^c f(x)\,dx - \int_0^e f(x)\,dx$ by refering to the figure with the indicated areas.



$2\int_a^c f(x)dx - \int_0^e f(x)dx$

$= 2[-A(A) + A(B) + A(C)] - [A(C) - A(D) + A(E)]$

$= 2(-1.12 + 3.32 + 2.05) - (2.05 - 3.46 + 3.25)$

$= \boxed{6.66}$

(a) 13.2

(b) 0

(c) 6.66

(d) -2.13

(e) None of these

6. The area of the region bounded by $y = 1 - x$, $y = 2$, $x = -4$, and $x = 1$ is

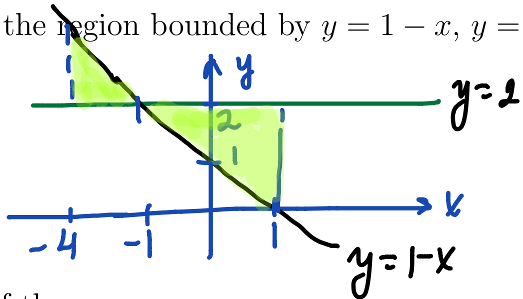
(a) 0

(b) -2

(c) 6.5

(d) 2.5

(e) None of these



Intersection: $1-x=2$
$x=-1$

$A = \int_{-4}^{-1}(1-x-2)\,dx + \int_{-1}^{1}[2-(1-x)]\,dx$
$= \boxed{6.5}$

7. The marginal revenue in dollars for selling boxes of Belgian Chocolates is

$$R'(x) = 20 - 0.54x + 0.048x^2$$

where x is the number of boxes sold. Find the revenue of selling 100 boxes if $R(0) = 0$.

(a) \$4,766

(b) \$10,463

(c) \$15,300

(d) \$23,412

(e) None of these

$R(x) = \int R'(x)\,dx = \int (20 - 0.54x + 0.048x^2)\,dx$

$= 20x - 0.54\frac{x^2}{2} + 0.048\frac{x^3}{3} + C$

$= 20x - 0.27x^2 + 0.016x^3 + C$

$0 = R(0) = C \implies C = 0$

$R(x) = 20x - 0.27x^2 + 0.016x^3$

$R(100) = 20(100) - 0.27(100^2) + 0.016(100^3)$
$= \boxed{15300}$

8. Evaluate the integral.

(a) *[3 points]* $\frac{1}{3}\int \frac{(3)5x^2}{(x^3-7)^4}dx$

$\left| \begin{array}{l} u = x^3 - 7 \\ du = 3x^2\,dx \end{array} \right| = \frac{1}{3}(5)\int \frac{du}{u^4} = \frac{5}{3}\int u^{-4}\,du$

$= \frac{5}{3}\,\frac{u^{-4+1}}{-4+1} + C = -\frac{5}{9}u^{-3} + C = \boxed{-\frac{5}{9}(x^3-7)^{-3} + C}$

(b) *[3 points]* $\int \frac{x+2}{\sqrt{1-x}}dx$

$\left| \begin{array}{l} u = 1 - x \\ du = -dx \\ x = 1 - u \end{array} \right|$

$= -\int \frac{1-u+2}{\sqrt{u}}\,du = -\int (3-u)u^{-1/2}\,du = -\int [3u^{-1/2} - u^{1/2}]\,du$

$= -3\int u^{-1/2}\,du + \int u^{1/2}\,du = -3\,\frac{u^{-1/2+1}}{-1/2+1} + \frac{u^{1/2+1}}{1/2+1} + C$

$= -6u^{1/2} + \frac{2}{3}u^{3/2} + C$

$= \boxed{-6(1-x)^{1/2} + \frac{2}{3}(1-x)^{3/2} + C}$