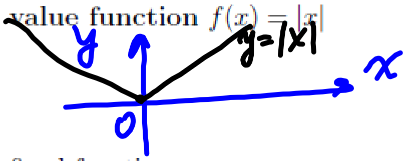
1.1 Increasing/Decreasing, Concavity, Continuity, and Piecewise-Defined Functions.

Piecewise-Defined Functions: Functions whose definitions involve more than one rule are called **piecewise-defined functions**. To graph, graph each rule over the appropriate portion of the domain.

An example of a piecewise-defined function is the **absolute value function** $f(x) = |x|$

$$f(x) = \begin{cases} -x, & x < 0, \\ x, & x \geq 0. \end{cases}$$



Example 1. Write $g(x) = |x+1|$ as an equivalent piecewise-defined function.

$$g(x) = \begin{cases} -(x+1) & x+1 < 0 \\ x+1, & x+1 \geq 0 \end{cases} = \begin{cases} -x-1, & x < -1 \\ x+1, & x \geq -1 \end{cases}$$

Example 2. For $f(x) = \begin{cases} x^2, & x \leq 0, \\ x+3, & x > 0. \end{cases}$

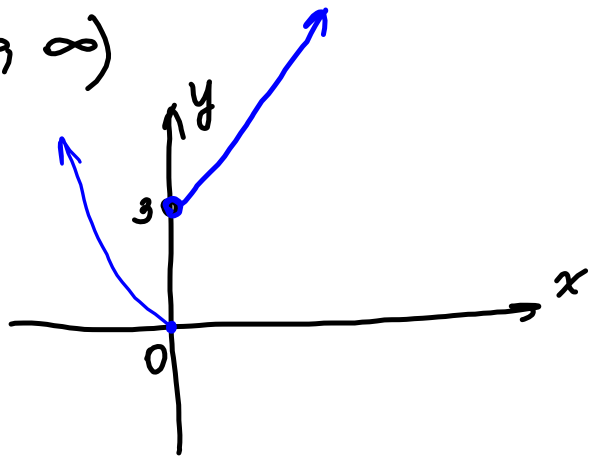
1. Find $f(-2) = (-2)^2 = 4$

   $f(0) = 0$

   $f(3). = 3+3 = 6$

2. Find the domain of $f(x)$. $(-\infty\ \infty)$

3. Make an accurate graph of $f(x)$.



Example 3. Find the domain of the function $f(x) = \begin{cases} \dfrac{x}{x+2}, & x < 0, \\ \dfrac{x+3}{x^2-9}, & x \geq 0. \end{cases}$

$x+2 \neq 0$, $x < 0$
$\boxed{x \neq -2}$

$x^2 - 9 \neq 0$, $x \geq 0$
$(x-3)(x+3) \neq 0$
$\boxed{x \neq 3}$ $x \neq -3$ drop
x should be positive

Domain: $\boxed{(-\infty, -2) \cup (-2, 3) \cup (3, \infty)}$

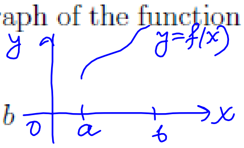
Example 4. A salesman receives a commission of \$1 per square yard for the first 500 yards of carpeting sold in a month and \$2 per yard for any additional carpet sold during the same month. If x is the number of yards of carpet sold and C is the commission, find C as a function of x.

$$\boxed{C(x) = \begin{cases} x, & 0 \leq x \leq 500 \\ 500 + 2(x - 500), & x > 500 \end{cases}}$$

Increasing, Decreasing, Concavity, and Continuity

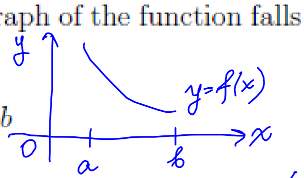
- A function is said to be **increasing** on the interval (a, b) if the graph of the function rises while moving left to right, or, equivalently, if

$$f(x_1) < f(x_2) \qquad \text{when} \qquad a < x_1 < x_2 < b$$



- A function is said to be **decreasing** on the interval (a, b) if the graph of the function falls while moving left to right, or, equivalently, if

$$f(x_1) > f(x_2) \qquad \text{when} \qquad a < x_1 < x_2 < b$$

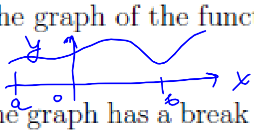


- If the graph of a function bends upward, we say that the function is **concave up**.

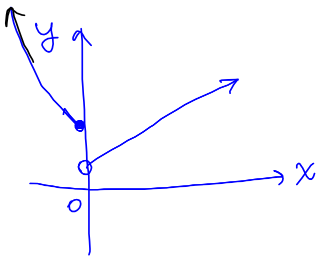
- If the graph of a function bends downward, we say that the function is **concave down**.

a line is neither concave up nor concave down

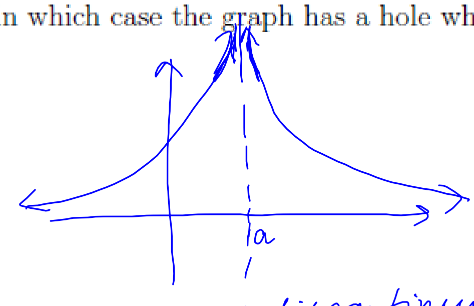
- A function is said to be **continuous** on an interval if the graph of the function does not have any breaks, gaps, or holes on that interval.



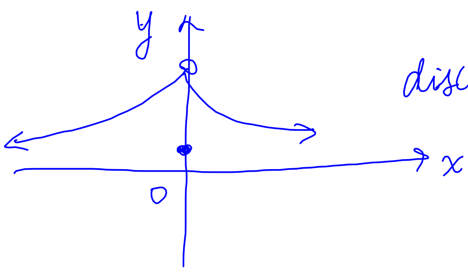
- A function is said to be **discontinuous** at a point c if the graph has a break or gap at the point $(c, f(c))$ or if $f(c)$ is not defined, in which case the graph has a hole where $x = c$.



discontinuous at $x=0$

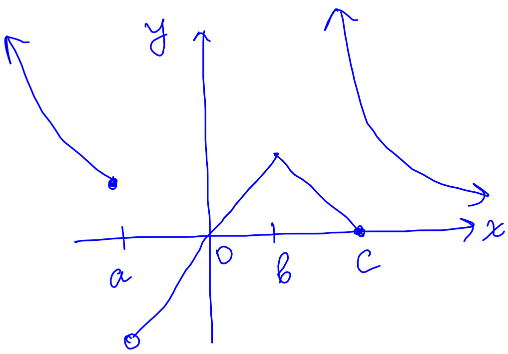


discontinuous at $x=a$



discontinuous at $x=0$

Example 5. Determine the interval(s) over which the following function is increasing, decreasing, concave up, concave down, and continuous.



Increasing: (a, b)

Decreasing: $(-\infty, a) \cup (b, c) \cup (c, \infty)$

Concave Up: $(-\infty, a) \cup (c, \infty)$

Concave Down: no

Continuous: $x \neq a$, $x \neq c$

$(-\infty, a) \cup (a, c) \cup (c, \infty)$