1.2 Break-Even Analysis and Market Equilibrium

Break-even analysis.

- Cost function $C(x) = mx + b$ =(variable cost)+(fixed cost)
- Price-demand function $p(x) = -cx + d$ (always decreasing) (p is a price of one unit if x units are sold)
- Supply equation $p = p(x)$ (gives the price p necessary for suppliers to make available x units for the market).
- Revenue function $R(x) = xp(x)$ =(price per unit)(number sold) (price-demand function)
- Profit function $P(x) = R(x) - C(x)$ =(revenue)-(cost)

Given a revenue function $R(x)$ and a cost function $C(x)$. **Break-even points** are the production levels at which

$$R(x) = C(x) \quad (\text{profit} = 0)$$

A *loss* occurs if $R(x) < C(x)$ and a *profit* occurs if $R(x) > C(x)$.

Example 1. A certain product has monthly fixed costs of \$360 and a variable cost of \$16 per item. The demand for the product is given by $p(x) = -24x + 400$.

1. Find the linear cost function.
   x is # of items

   $C(x)$ = fixed costs + variable cost
   $= 360 + 16x$ – cost function

2. Find the revenue and profit functions.

   $R(x) = x\,p(x) = x(-24x+400)$
   $= -24x^2 + 400x$ – revenue function

   $P(x) = R(x) - C(x) = -24x^2 + 400x - (360 + 16x)$
   $= -24x^2 + 384x - 360$ – profit function ($a = -24$, $b = 384$)

3. Find the value of x that maximizes profit.

   $$x_{max} = -\frac{b}{2a} = -\frac{384}{2(-24)} = \boxed{8}$$

4. Find the break-even points. $P(x) = 0$

   $$-24x^2 + 384x - 360 = 0$$
   $$-24(x^2 - 16x + 15) = 0$$
   $$-24(x-1)(x-15) = 0$$
   $$\boxed{x_1 = 1 \quad x_2 = 15}$$

Market equilibrium.

The point at which supply equals demand is called the **equilibrium point**. The x-coordinate of the equilibrium point is called the **equilibrium quantity** and the p-coordinate is called the **equilibrium price**.

Example 2. For a certain product, 1000 units can be sold at a price of \$20.00 per unit. For each increase of \$2.00 in the price, 100 fewer can be sold. Suppliers will provide only 670 units if the price is \$20.00 and will provide none if the price is \$13.30 or lower. Find the equilibrium quantity and price.

1) demand equation.

$p(x) = mx + b$ — linear   passes through $(\overbrace{1000}^{x_0}, \overbrace{20}^{p_0})$

slope $m = \frac{\Delta p}{\Delta x} = \frac{2}{-100} = -0.02$

$p(x) - \overset{20}{p_0} = \overset{-0.02}{m}(x - \overset{1000}{x_0})$

$p(x) - 20 = -0.02(x - 1000)$

$p(x) - 20 = -0.02x + 20$

$\boxed{p(x) = -0.02x + 40}$ demand equation

2) supply equation. $p(x) = kx + n$ linear function.   should pass through $(670, 20)$ and $(\overset{x_0}{0}, \underbrace{13.30}_{p_0})$

slope $k = \frac{\Delta p}{\Delta x} = \frac{20 - 13.30}{670 - 0} = \frac{6.7}{670} = 0.01$

$p(x) - \overset{13.30}{p_0} = \overset{0.01}{k}(x - \overset{0}{x_0})$

$p(x) - 13.30 = 0.01x$

$\boxed{p(x) = 0.01x + 13.30}$ supply equation

3) equilibrium

$-0.02x + 40 = 0.01x + 13.30$

$0.03x = 40 - 13.30$

$0.03x = 26.7$

$x = \frac{26.7}{0.03} = 890 \text{ (units)}$ equilibrium quantity

$p = 0.01(890) + 13.30 = \$22.20$ equilibrium price