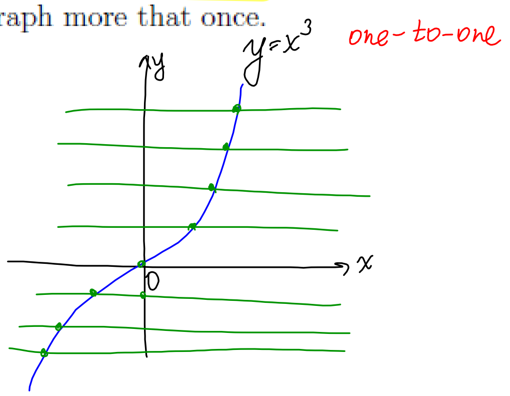
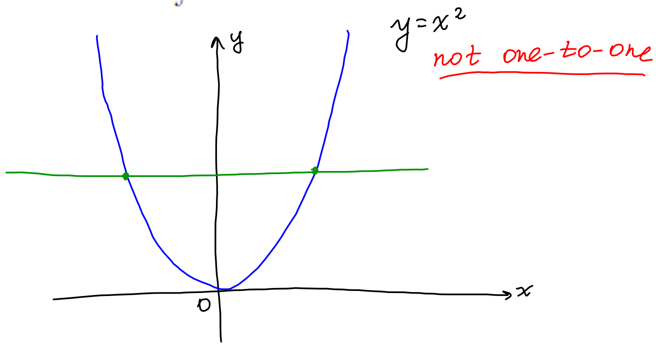
Section 1.5 **Logarithms**

Inverse functions.

A function f is said to be one-to-one if each range value corresponds to exactly one domain value.

Horizontal line test: A function is one-to-one if and only if no horizontal line intersects its graph more that once.

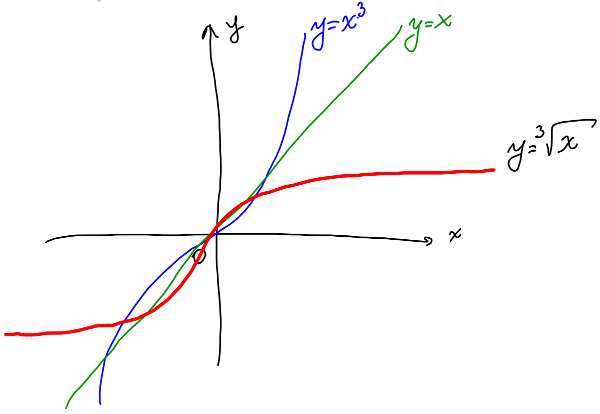




If f is a one-to-one function, then the **inverse** of f is the function formed by interchanging the independent and dependent variables for f .

$y=x^3$, ~~its~~ its inverse is $x=y^3$, then solve for y: $\boxed{y=\sqrt[3]{x}}$

Note: If f is one-to-one, then the graph of its inverse, g, is a reflection about the line $y = x$. If f is not one-to-one, then f does not have an invers.

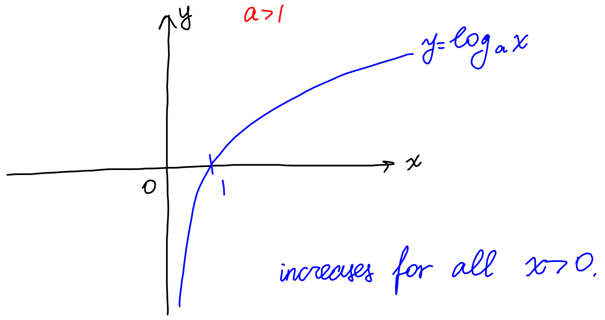


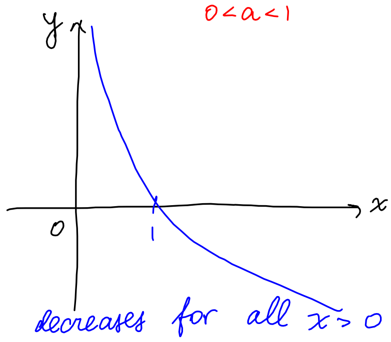
Logarithms.

$$\log_a x = y \iff a^y = x \;, \quad a>0,\ a\neq 1$$

y is the power to which you should raise a to get x

- Function $f(x) = \log_a x$ is one-to-one continuous function with domain $(0,\infty)$ and range $(-\infty,\infty)$.
- If $a > 1$, then $f(x) = \log_a x$ is increasing function
- If $0 < a < 1$, then $f(x) = \log_a x$ is decreasing function





The cancellation equations

$$\boxed{\log_a a^x = x} \quad \boxed{a^{\log_a x} = x}$$

$\log_2(2^3)=3$

$\log_9(9^2)=2$

$\log_5(5^{-3})=-3$

$2^{\log_2 4}=4$

$4^{\log_4 5}=5$

$142^{\log_{142}(504)}=504$

If $a = e$, then

$$\log_e x = \ln x$$

natural log

$$\boxed{\ln e^a = a, \quad e^{\ln b} = b}$$

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If $a = 10$, then

$$\log_{10} x = \log x$$

$$\boxed{\log 10^a = a, \quad 10^{\log b} = b}$$

Example 1. Evaluate:

1. $\log_2 64 = \log_2(2^6) = \boxed{6}$

2. $\log_6 \frac{1}{36} = \log_6(6^{-2}) = \boxed{-2}$

3. $2^{\log_2 3 + \log_2 5} = 2^{\log_2 3} \cdot 2^{\log_2 5} = (3)(5) = \boxed{15}$

Example 2. Find the domain and the range for the function $f(x) = 2 + \ln(2x+1)$.

- Domain. $2x+1 > 0$

  $x > -\frac{1}{2}$

  $\boxed{\left(-\frac{1}{2}, \infty\right)}$

- Range: $\boxed{(-\infty, \infty)}$

Properties of logarithms

If $x, y > 0$ and k is a constant, then

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^k = k \log_a x \qquad \log_a \left(\frac{1}{x}\right) = -\log_a x$
4. $\log_{a^k} x = \frac{1}{k} \log_a x \qquad \log_{\frac{1}{a}} x = -\log_a x$
5. $\log_a a = 1$
6. $\log_a 1 = 0$
7. $\log_a x = \frac{\log_b x}{\log_b a}$

Example 3. Evaluate $e^{3\ln 2-1}\ln(5e^2)$

$\boxed{e^{\ln a}=a}$

- $e^{3\ln 2-1}=e^{3\ln 2}\cdot e^{-1}=(e^{\ln 2})^3\cdot e^{-1}=(2)^3e^{-1}=8e^{-1}$
- $\ln(5e^2)=\ln 5+\ln(e^2)=\ln 5+2$

$\ln(e^6)=6$

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- answer $\boxed{8e^{-1}(\ln 5+2)}$

Example 4. Express the given quatities as a single logarithm:

1. $\log_2 x+5\log_2(x+1)+\frac{1}{2}\log_2(x-1)$

   $=\log_2 x+\log_2(x+1)^5+\log_2(x-1)^{1/2}$

   convert sum into the product

   $=\boxed{\log_2\left[x(x+1)^5(x-1)^{1/2}\right]}$

2. $2\ln 4-\ln 2$

   $=\ln(4^2)-\ln 2$

   convert the difference into the quotient

   $=\ln\frac{4^2}{2}$

   $=\boxed{\ln 8}$

3. $\log_8 a-\log_4 b+\log_2 c$

   $=\log_{2^3}a-\log_{2^2}b+\log_2 c$

   $=\frac{1}{3}\log_2 a-\frac{1}{2}\log_2 b+\log_2 c$

   $=\log_2(a^{1/3})-\log_2(b^{1/2})+\log_2 c$

   $=\boxed{\log_2\frac{a^{1/3}c}{b^{1/2}}}$

Example 5. Solve the equation:

1. $\log_{25} x = \frac{1}{2}$

$x = 25^{1/2}$

$= \boxed{5}$

2. $\log_2(x-1) = 2 + \log_2(x-4)$

$\log_2(x-1) - \log_2(x-4) = 2$

$\log_2 \frac{x-1}{x-4} = 2$

$\frac{x-1}{x-4} = 2^2$

$\frac{x-1}{x-4} = 4$

$x - 1 = 4(x-4)$

$x - 1 = 4x - 16$

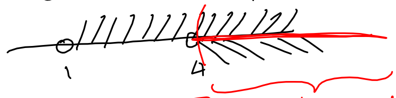
$3x = 15$

$\boxed{x = 5}$ in the domain.

Domain:

$x - 1 > 0 \qquad x > 1$

$x - 4 > 0 \qquad x > 4$



intersection:

Domain is $\boxed{(4, \infty)}$

3. $1.03^x = 2.43$

$\log(1.03^x) = \log(2.43)$

$x \log(1.03) = \log(2.43)$

$x = \frac{\log(2.43)}{\log(1.03)} \approx \boxed{30.04}$

Applications.

$\boxed{\ln e = 1}$

Example 6. How long will it take for the amount in an account to double if the money is compounded continuously at an annual interest rate of 4.7%?

$A = Pe^{rt}$

A - future value

P - present value

$r = 4.7\% = 0.047$

find t such that $A = 2P$

$2\cancel{P} = \cancel{P} e^{0.047t}$

$2 = e^{0.047t}$

$\ln(2) = \ln(e^{0.047t})$

$\ln(2) = 0.047t \cancel{\ln e}$ 1

$0.047t = \ln 2$

$t = \frac{\ln 2}{0.047} \approx \boxed{15}$

Example 7. Suppose inflation causes the value of a dollar to decrease by 3% a year. How long does it take for a dollar to be worth \$0.75?

$A = P(1-r)^t$, $r = 3\% = 0.03$

$A = 0.75$

$P = 1$

$0.75 = 1(1 - 0.03)^t$

$0.75 = (0.97)^t$

$\log(0.75) = \log((0.97)^t)$

$\log(0.75) = t \log(0.97)$

$t = \frac{\log(0.75)}{\log(0.97)} \approx \boxed{9}$