Section 5.1. **First derivative.**

The values of x in the domain of f where $\boxed{f'(x) = 0}$ or where $\boxed{f'(x) \text{ does not exist}}$ are called the critical values of f.

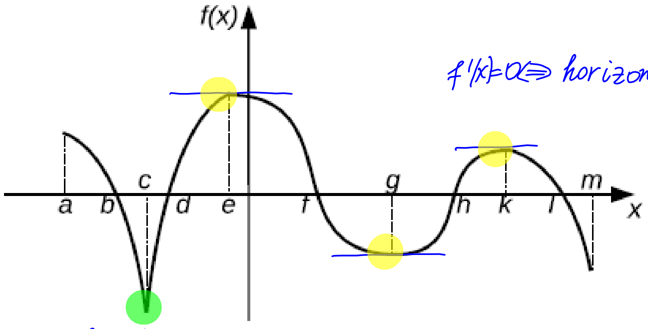
The function f is increasing on an interval (a, b) if $\boxed{f(x_2) > f(x_1)}$ whenever $a < \boxed{x_1 < x_2} < b$, and f is decreasing on (a, b) if $\boxed{f(x_2) < f(x_1)}$ whenever $a < \boxed{x_1 < x_2} < b$.

Increasing / decreasing test

- If $\boxed{f'(x) > 0}$ on an interval, then f is increasing on that interval
- If $\boxed{f'(x) < 0}$ on an interval, then f is decreasing on that interval

Example 1. Given the graph of the function f.

$f'(x)=0 \Rightarrow$ horizonlat tangent



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1. Identify intervals on which $f'(x) > 0$. What does it say about $f(x)$?

   $f'(x)>0$ (f increases): $\boxed{(c,e)\cup(g,k)}$

2. Identify intervals on which $f'(x) < 0$. What does it say about $f(x)$?

   $f'(x)<0$ (f decreases): $\boxed{(a,c)\cup(e,g)\cup(k,m)}$

3. What are the x-coordinate(s) of the points where $f'(x)$ does not exist? Where $f'(x) = 0$? What does it say about $f(x)$?

   $f'(e)=f'(g)=f'(k)=0$

   $f'(c)$ DNE

$f(c)$ - local max

$f(c)$ - local min

A function f has a local maximum at c if $f(c) \geq f(x)$ when x is near c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ when x is near c. The quantity $f(c)$ is called a local extremum if it either a local maximum or a local minimum.

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If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $\boxed{f'(c) = 0}$

The first derivative test Suppose that c is a critical number of a continuous function f.

- If f' changes from positive to negative at c, then f has a local max at c. f': $+$ c $-$ x; c - local max
- If f' changes from negative to positive at c, then f has a local min at c. f': $-$ c $+$ x; c - local min
- If f' does not change sign at c, then f has a no local max or min at c. f': $+$ c $+$ x; f': $-$ $-$ x; neither

To find intervals on which a function is increasing or decreasing, we will construct a sign chart for $f'(x)$ to determine which values of x make $f'(x) > 0$ and which values make $f'(x) < 0$.

Example 2. Find the critical values of f, the intervals on which f is increasing, the intervals on which f is decreasing, and the local extrema for the following functions. Do not graph.

1. $f(x) = x^3 + 3x^2 - 9x + 3$

Step 1. Find the critical values of $f(x)$.

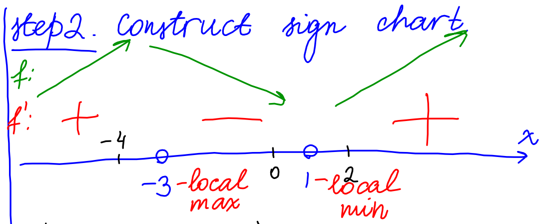
$$f'(x) = \frac{3x^2+6x-9}{3} = \frac{0}{3}$$

$$x^2+2x-3=0$$

$$(x+3)(x-1)=0$$

$$x_1=-3,\ x_2=1$$

Step 2. Construct sign chart



$$f'(-4) = (-4)^2+2(-4)-3 = 5 > 0$$

$$f'(0) = -9 < 0$$

$$f'(2) = 2^2+2(2)-3 = 5 > 0$$

f increases on: $(-\infty,-3)\cup(1,\infty)$
f decreases on: $(-3,1)$
f has a local max @ $x=-3$
f has a local min @ $x=1$

2. $f(x) = \dfrac{x^2}{x-1}$

Critical values:

$$f'(x) = \frac{(x^2)'(x-1) - x^2(x-1)'}{(x-1)^2} = \frac{2x(x-1)-x^2}{(x-1)^2}$$

$$= \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = 0$$

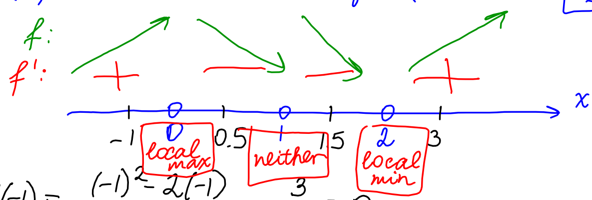
$$x^2-2x=0$$

$$x(x-2)=0$$

$$\boxed{x_1=0,\ x_2=2}$$

$f'(x)$ does not exist if $(x-1)^2=0$ or $\boxed{x_3=1}$

Sign chart:



$$f'(-1) = \frac{(-1)^2-2(-1)}{(-1-1)^2} = \frac{3}{4} > 0$$

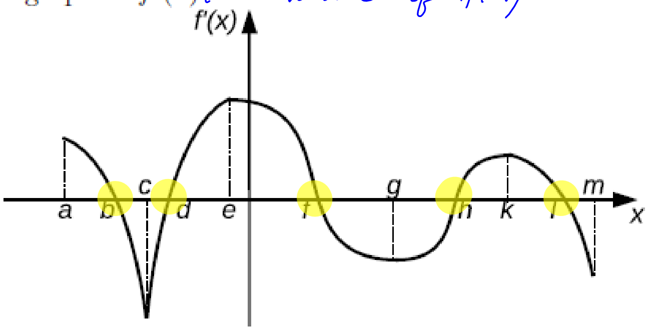
$$f'(0.5) = \frac{(0.5)^2-2(0.5)}{(0.5-1)^2} = \frac{-0.75}{0.25} < 0$$

$$f'(1.5) = \frac{(1.5)^2-2(1.5)}{(1.5-1)^2} = \frac{2.25-3}{(0.5)^2} = \frac{-0.75}{0.25} < 0$$

$$f'(3) = \frac{3^2-3(2)}{(3-1)^2} = \frac{3}{4} > 0$$

f increases on $(-\infty,0)\cup(2,\infty)$
f decreases on $(0,1)\cup(1,2)$
f has a local max @ $x=0$
f has a local min @ $x=2$

Example 3. Given the graph of $f'(x)$ (derivative of $f(x)$)



1. Identify intervals on which f is increasing/decreasing.

   f increases on $\boxed{(a,b)\cup(d,f)\cup(h,l)}$ $(f'(x)>0)$

   f decreases on $\boxed{(b,d)\cup(f,h)\cup(l,m)}$ $(f'(x)<0)$

2. Identify the x coordinates of the points where f has a local maximum/local minimum.

   f has local max @ $x=b$, $x=f$, $x=l$

   f has local min @ $x=d$, $x=h$