Section 5.4 **Additional curve sketching.**

Graphing strategy.

1. Analyze $f(x)$.
   (a) Find the domain of f.
   (b) Find the intercepts.
   (c) Find asymptotes.
2. Analyze $f'(x)$.
   (a) Find all critical value of $f(x)$.
   (b) Construct a sign chart for $f'(x)$.
   (c) Determine the intervals on which f is increasing and decreasing.
   (d) Find local maxima and minima.
3. Analyze $f''(x)$.
   (a) Find all value af x such that $f''(x) = 0$.
   (b) Construct a sign chart for $f''(x)$.
   (c) Determine the intervals on which f is concave upward and concave downward.
   (d) Find inflection points.
4. Sketch the graph of f.

$3-2x-x^2>0$

$(1-x)(x+3)>0$ $-\ \ +\ \ -$ at -3, 1

Example. Using calculus, sketch

1. $f(x)=\dfrac{1}{3-2x-x^2}$. $=(3-2x-x^2)^{-1}$

Domain: $3-2x-x^2\neq 0$

$(1-x)(x+3)\neq 0$

$x\neq -3,\ x\neq 1$

domain: $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$

range: $(-\infty,\infty)$

NO intercepts.

asymptotes: vertical: $x=-3$, $x=1$

$\lim\limits_{x\to\infty}\dfrac{1}{3-2x-x^2}=0$ horizontal asymptote: $y=0$

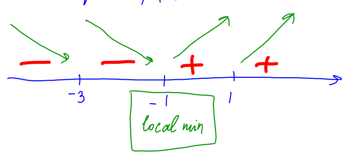
$f'(x)=(-1)(3-2x-x^2)^{-2}\underbrace{(3-2x-x^2)'}_{-2-2x}$

$=(-1)(-2-2x)(3-2x-x^2)^{-2}$

$=(2+2x)(3-2x-x^2)^{-2}=0$

$2+2x=0$ or $\boxed{\begin{matrix}x=-1\\ x=-3,\ x=1\end{matrix}}$ critical points

sign chart for $f'(x)$:



$f'(-4)=-5(25)^{-1}<0$

$f'(-2)=-2(9)^{-1}<0$

$f'(0)=2(9)^{-1}>0$

$f'(2)=6(25)^{-1}>0$

f decreases on $(-\infty,-3)\cup(-3,-1)$

f increases on $(-1,1)\cup(1,\infty)$

f has a local min @ $x=-1$ $f(-1)=\dfrac{1}{3-2(-1)-(-1)^2}=\dfrac{1}{4}$

$f''(x)=2(3-2x-x^2)^{-2}+(2+2x)(-2)(3-2x-x^2)^{-3}(3-2x-x^2)'$

$=2(3-2x-x^2)^{-2}+(2+2x)(-2)(3-2x-x^2)^{-3}(-2-2x)$

$=\dfrac{2}{(3-2x-x^2)^2}+\dfrac{(-2)(2+2x)(-2-2x)}{(3-2x-x^2)^3}=0$

$=\dfrac{2(3-2x-x^2)+(-2)(-4-4x-4x-4x^2)}{(3-2x-x^2)^3}$

$=\dfrac{6-4x-2x^2+8+16x+8x^2}{(3-2x-x^2)^3}$

$=\dfrac{6x^2+12x+14}{(3-2x-x^2)^3}=0$ (denominator $\neq 0$ $x\neq-3,\ x\neq 1$)

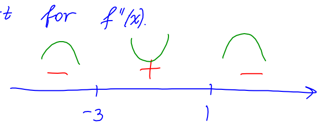
$\dfrac{6x^2+12x+14}{2}=\dfrac{0}{2}$

$3x^2+6x+7=0$

$x_1=\dfrac{-6+\sqrt{36-4(7)(3)}}{6}=\dfrac{-6+\sqrt{36-84}}{6}$

NO INFLECTION POINTS.

sign chart for $f''(x)$.



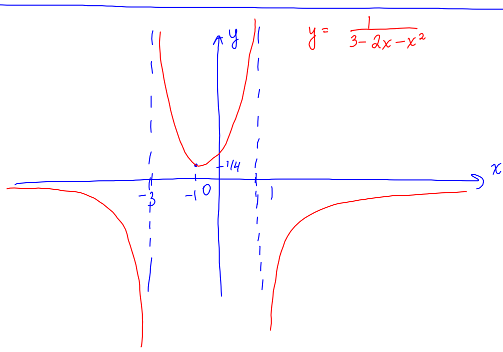
$f''(-4)=\dfrac{6(16)+12(-4)+14}{(3+8-16)^3}=\dfrac{62}{-125}<$

$f''(0)=\dfrac{14}{27}>0$

$f''(2)=\dfrac{6(4)+12(2)+14}{(3-4-4)^3}=\dfrac{62}{-125}<$

f is CU on $(-\infty,-3)\cup(1,\infty)$

f is CD on $(-3,1)$



$y=\dfrac{1}{3-2x-x^2}$

2. $f(x) = xe^{2x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

$xe^{2x} > 0$

since $e^{2x} > 0$ for all x,

then $\boxed{f(x) > 0 \text{ when } x > 0}$

$\boxed{f(x) < 0 \text{ when } x < 0}$

intercepts.

x-intercepts.

solve $xe^{2x} = 0$

$e^{2x} \neq 0$, thus $x = 0$

$\boxed{(0,0)}$

y-intercept: $y = 0e^{2(0)} = 0$

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asymptotes: $\boxed{\text{no vertical asymptotes}}$

$\lim_{x\to\infty} xe^{2x} = \infty$

$\lim_{x\to-\infty} xe^{2x} = 0 \quad \left(\lim_{x\to-\infty} e^{2x} = 0 \right)$

$\boxed{y=0 \text{ is the horizontal asymptote when } x\to-\infty \text{ only}}$ (in one direction only)

$f'(x) = (x)'e^{2x} + x(e^{2x})' = e^{2x} + xe^{2x}(2x)' = e^{2x} + xe^{2x}(2)$

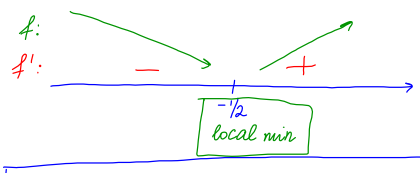
$= e^{2x} + 2xe^{2x} = 0$

$e^{2x}(1+2x) = 0$

$e^{2x} \neq 0 \qquad 1+2x = 0$

$x = -\frac{1}{2}$

sign chart for $f'(x)$



$f'(-1) = e^{-2}(1-2) \approx -.14 < 0$

$f'(0) = e^{0}(1) = 1 > 0$

f increases on $(-\frac{1}{2}, \infty)$

f decreases on $(-\infty, -\frac{1}{2})$

f has a local min @ $x = -\frac{1}{2}$. $f(-\frac{1}{2}) = -\frac{1}{2}e^{-2\frac{1}{2}} \approx -.18$

$f''(x) = (e^{2x} + 2xe^{2x})' = 2e^{2x} + (2x)'e^{2x} + 2x(e^{2x})'$

$= 2e^{2x} + 2e^{2x} + 2x\,2e^{2x}$

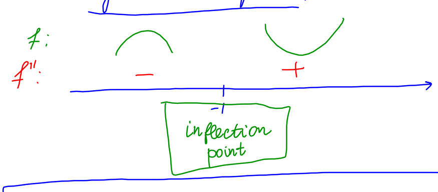
$= 4e^{2x} + 4xe^{2x} = 0$

$e^{2x}(4+4x) = 0$

$e^{2x} \neq 0 \qquad 4+4x = 0$

$x = -1$

sign chart for f'':



$f''(0) = 4e^{0} = 4 > 0$

$f''(-2) = e^{-4}(4-8) \approx -0.73 <$

f is CU on $(-1, \infty)$

f is CD on $(-\infty, -1)$

f has the inflection point @ $x = -1$. $f(-1) = -1e^{-2} \approx -.14$