Table of indefinite integrals.

1. $\int a\ dx = ax + C$, a is a constant

2. $\displaystyle\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

3. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

4. $\int \mathrm{e}^x dx = \mathrm{e}^x + C$

Properties of indefinite integrals.
For k a constant,

1. $\int k f(x) dx = k \int f(x) dx$

2. $\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$

Section 6.2 **Substitution.**

Reversing the chain rule. $[f(g(x))]' = f'(g(x))\,g'(x)$

$$\boxed{\int f'(g(x))g'(x)dx = f(g(x)) + C}$$

General indefinite integral formulas.

1. $\boxed{\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C, \quad n \neq -1.}$
2. $\boxed{\int e^{f(x)} f'(x)dx = e^{f(x)} + C}$
3. $\boxed{\int \frac{1}{f(x)} f'(x)dx = \ln|f(x)| + C.}$

Example 1. Find each indefinite integral.

1. $\int (\underbrace{5x+6}_{u})^{11} \underbrace{dx}_{\frac{1}{5}du}$

$u = 5x+6 \Rightarrow$ solve for x: $5x = u-6$

$x = \frac{u-6}{5}$

$x = \frac{u}{5} - \frac{6}{5} = x(u)$

$x'(u) = \frac{1}{5}$

$dx = x'(u)du$

$dx = \frac{1}{5}du$

$$= \int u^{11} \frac{1}{5} du = \frac{1}{5} \int u^{11} du = \frac{1}{5} \frac{u^{11+1}}{11+1} + C$$

$$= \frac{1}{5} \frac{u^{12}}{12} + C$$

$$= \boxed{\frac{1}{60}(5x+6)^{12} + C}$$

2. $\int \underbrace{3x^2}_{du} \sqrt[4]{\underbrace{x^3+1}_{u}}\,dx$

$u = x^3+1$, $(x^3+1)' = 3x^2$

$du = u'(x)dx$

$du = 3x^2 dx$

$$= \int \sqrt[4]{u}\,du = \int u^{1/4} du = \frac{u^{1/4+1}}{1/4+1} + C = \frac{u^{\frac{5}{4}}}{\frac{5}{4}} + C = \frac{4}{5}u^{\frac{5}{4}} + C$$

$$= \boxed{\frac{4}{5}(x^3+1)^{5/4} + C}$$

3. $\frac{1}{4}\int \frac{\overbrace{4(x^3+1)\,dx}^{du}}{(\underbrace{x^4+4x}_{u})^6}$

$$
\begin{aligned}
u &= x^4+4x \\
u' &= (x^4+4x)' \\
&= 4x^3+4 = 4(x^3+1) \\
du &= u'(x)\,dx \\
du &= 4(x^3+1)\,dx
\end{aligned}
$$

$$
\begin{aligned}
= \frac{1}{4}\int \frac{du}{u^6} = \frac{1}{4}\int u^{-6}\,du &= \frac{1}{4}\,\frac{u^{-6+1}}{-6+1} + C \\
&= \frac{1}{4}\,\frac{u^{-5}}{-5} + C \\
&= \boxed{-\frac{1}{20}(x^4+4x)^{-5} + C}
\end{aligned}
$$

4. $-\int e^{\overset{u}{4-x}}\,\overbrace{dx(-1)}^{du}$

$$
\begin{aligned}
u &= 4-x \\
du &= (4-x)'\,dx \\
du &= (-1)\,dx
\end{aligned}
$$

$$
\begin{aligned}
= -\int e^u\,du &= -e^u + C \\
&= \boxed{-e^{4-x}+C}
\end{aligned}
$$

5. $\frac{1}{2}\int \underbrace{2x\,dx}_{du}\, e^{\overset{u}{x^2}}$

$$
\begin{aligned}
u &= x^2 \\
du &= 2x\,dx
\end{aligned}
$$

$$
\begin{aligned}
= \frac{1}{2}\int e^u\,du &= \frac{1}{2}e^u + C \\
&= \boxed{\frac{1}{2}e^{x^2} + C}
\end{aligned}
$$

6. $-\frac{1}{2}\int \frac{-2x\,dx}{\underbrace{2-x^2}_{u}}$ $= du$

$u = 2 - x^2$

$du = -2x\,dx$

$$= -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| + C$$

$$= \boxed{-\frac{1}{2}\ln|2-x^2| + C}$$

7. $\int \overset{u-4}{x}\,(\overset{u}{x+4})^5\,\overset{du}{dx}$

$u = x+4$ solve for x: $x = u-4$

$du = dx$

$$= \int (u-4)u^5\,du = \int (u^6 - 4u^5)\,du = \frac{u^{6+1}}{6+1} - 4\frac{u^{5+1}}{5+1} + C$$

$$= \frac{u^7}{7} - 4\frac{u^6}{6} + C$$

$$= \boxed{\frac{(x+4)^7}{7} - \frac{4(x+4)^6}{6} + C}$$